A

KEY

TO THE

NATIONAL ARITHMETIC,

EXHIBITING THE OPERATION OF

THE MORE DIFFICULT QUESTIONS

IN THAT WORK;

FOR THE USE OF TEACHERS ONLY.

By BENJAMIN GREENLEAF, A. M.,

AUTHOR OF THE "COMMON SCHOOL ARITHMETIC," "ALGEBRA," ETC.

NEW ELECTROTYPE EDITION.

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1859.

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☞ Two editions of the NATIONAL ARITHMETIC, and also of the COMMON SCHOOL ARITHMETIC, one containing the ANSWERS to the examples, and the other without them, are published. Teachers are requested to state in their orders *which edition* they prefer.

PREFACE.

THE object of the author, in this publication, is to aid the teacher in communicating instruction to his pupils, and in detecting any error which they may have made in the operation of the examples.

Every instructor who has a large number of scholars under his care is aware that it is a great tax on his time, especially when in school, to examine the operation of many arithmetical questions; whereas, by the aid of a Key, he may readily detect any mistake in the operation. Besides, amid the labors of the school-room, it is often very difficult for the most able arithmetician to recollect, at the moment, all the principles involved in the solution of difficult questions; but, by recurring to a Key, this difficulty will be obviated.

The author would recommend to teachers never to point out *directly* to the pupil the method of solving a problem, nor perform the labor for him; but suggest and explain such principles as will enable him to perform the question himself.

The answers to all the examples in the Arithmetic are inserted in the Key, for the convenience of those teachers who may prefer to use the edition of the Arithmetic which does not contain the answers.

B. GREENLEAF.

BRADFORD, MASS., *September*, 1857.

CONTENTS.

KEY

TO

GREENLEAF'S ARITHMETIC.

NUMERATION.

1.	(Art. **40, p. 22.**)	29
2.		407
3.		23,007
4.		5,000,027
5.		.7,205,005
6.		2,207,604,009
7.		105,909,308,201
8.		9,000,000,008,000,000,046
9.		15,000,000,000,031,000,017
10.		507,000,000,000,203,000,057,000,018
11.		9,000,000,000,000,000,047,007,002,000,392
12.		15,000,000,000,000,000,000,000,000,010,127,026,320,426

Note. — The above is the French method.

ADDITION.

(Art. **47, p. 26.**)

17.	30,530	24.	300,000	31.	3,837,156
18.	31,643	25.	264,088	32.	150
19.	26,798	26.	357,477	33.	26,199
20.	28,578	27.	276,605	34.	264
21.	34,383	28.	3,980,839	35.	4,801,393
22.	29,340	29.	4,183,478	36.	5,067,696
23.	283,649	30.	31,881,050	37.	5,640,426

1*

38.	4,344,737	42.	62,075	46.	119
39.	4,935,497	43.	10,601	47.	$ 228
40.	1,937,678	44.	11,087	48.	$ 37,443
41.	118,106	45.	$ 82,871		

(ART. 48, p. 28.)

| 2. | 296 | 4. | 25,976 | 6. | 936,318 |
| 3. | 1,832 | 5. | 643,322 | 7. | 23,191,876 |

SUBTRACTION.

4. (ART. 52, p. 31.)	5,676	24.	408,881,883,715
5.	5,119	25.	61,475,423
6.	4,409	26.	999,999
7.	2,589	27.	1
8.	48,447	28.	6,686,136
9.	46,698	29.	760,702,380
10.	17,672	30.	31,309,891
11.	53,859	31.	16,680,605
12.	411,001	32.	10,014,098,379
13.	426,944	33.	85
14.	6,202,102	34.	110
15.	799,081	35.	190
16.	1,439	36.	993,044
17.	92,690	37.	$ 11,810
18.	243,334	38.	
19.	617,441	39.	173
20.	900,981	40.	1,026
21.	98,999,080	41.	4,004
22.	788,889	42.	45 and 38
23.	9,393,239,896,461	43.	519,853,026

1. (ART. 53, p. 33.)	138	5.	1,237,311
2.	525 dollars.	6.	2,500,000
3.	389 dollars.	7.	49,632 dollars.
4.	9,187 dollars.	8.	572,206 dollars.

MULTIPLICATION.

5. (ART. **63**, p. **40.**)	3,156,492	26.	59,784
6.	6,172,835	27.	3,545,304
7.	1,979,796	28.	584,720,181,340
8.	30,316,704	29.	594,731,545
9.	16,294,896	30.	119,109,094,835
10.	22,204,188	31.	406,781,410,014
11.	45,005,091	32.	318,697,622,634
12.	77,377,566	33.	230,896,467,247
13.	293,468,329	34.	137,260,338,494
14.	161,539,842	35.	213,255,462,816
15.	274,135,320	36.	395,018,272
16.	17,247,986,832	37.	70,136,114,040
17.	$26,645	38.	475,065,601,586
18.	$5,529	39.	204,060,808,060,402
19.	$2,779	40.	915,527,086,788,307
20.	$21,053	41.	454,115,186,861,492
21.	13,505	42.	12,032,109,124,168,023
22.	24,386	43.	81,000,108,000,036
23.	$4,886	44.	52,370,625
24.	4,888	45.	114,972
25.	9,021	46.	29,657,416,470,704

(ART. **64**, p. **42.**)

2.	252,801	5.	2,639,559,272
3.	$11,025	6.	897,264
4.	2,784		

(ART. **64**, p. **43.**)

3.	18,190	8.	63,126,063,000
4.	410,600	9.	3,720
5.	70,000,000	10.	$888,000
6.	9,594,000,000	11.	$2,050,000
7.	700,000,000	12.	2,850,000,000

DIVISION.

	(Art. 77, p. 49.) Quotients.	Rem.			Quotients.	Rem.
			30.		17,327	
8.	25,569	2	31.		69,255	1
9.	151,617	2	32.		71,451	
10.	66,930	2	33.		8,650	111
11.	12,090,447	2	34.		90,365	28
12.	20,747	8	35.		4,598	297
13.	39,936	4	36.		226,447	174
14.	260,171	28	37.		5,091	5091
15.	68,241	80	38.		7,060,504	4267
16.	111,946,492	1	39.		88,888	2341
17.	23,762,387	2	40.		800,008	4567
18.	20,166,474	1	41.		908,007,004	8765
19.	17,964,186	4	42.		2,069	
20.	33,081,425	3	43.		2,700 pounds.	
21.	13,698,246	4	44.		134	
22.	26,316,692	1	45.		987	
23.	169,739,167	3	46.		17	
24.	133,557,795	1	47.		$85\frac{148}{185}$	
25.	129,629,629	3	48.		$384\frac{8}{13}$ hours.	
26.	126,984,126	6	49.		$12,402	
27.	17,166	0	50.		35	
28.	153,227	44	51.		A's 76; B's 68; C's 48.	
29.	275,175	0				

	(Art. 78, p. 51.)					
3.		321		7.	138	45
4.		308	1	8.	273	18
5.		38	38	9.	$121\frac{19}{45}$	
6.		507	40			

	(Art. 79, p 52.)					
				8.	37	411,111
3.	12,345,678		9	9.	89,765	432,156
4.	9,876,543		0	10.	164,000	
5.	5		2,100	11.	21.42\frac{4}{9}$	
6.	11		91,858	12.	$494\frac{143}{133}$	
7.	3		137,851			

CANCELLATION.

(Art. 85, p. 55.)

3. $\dfrac{\overset{2}{\cancel{24}} \times 16}{\underset{1}{\cancel{12}}} = 32.$

4. $\dfrac{\overset{3}{\cancel{48}} \times 8}{\underset{1}{\cancel{16}}} = 24.$

5. $\dfrac{\overset{1}{\cancel{7}} \times \overset{5}{\cancel{10}} \times \overset{2}{\cancel{12}} \times 5}{\underset{2}{\cancel{14}} \times \underset{9}{\cancel{18}} \times \cancel{6}} = 2\frac{5}{9} = 2\frac{7}{9}.$

6. $\dfrac{15 \times \overset{1}{\cancel{7}} \times \overset{1}{\cancel{27}} \times \overset{4}{\cancel{40}}}{\underset{2}{\cancel{54}} \times \underset{2}{\cancel{14}} \times \underset{1}{\cancel{10}} \times 2} = \frac{15}{2} = 7\frac{1}{2}.$

7. $\dfrac{\overset{1}{\cancel{13}} \times \overset{5}{\cancel{15}} \times \overset{2}{\cancel{20}} \times 5}{\underset{2}{\cancel{26}} \times \cancel{10} \times \cancel{2} \times \cancel{3}} = \frac{25}{2} = 12\frac{1}{2}.$

8. $\dfrac{\overset{4}{\cancel{28}} \times \overset{1}{\cancel{27}} \times \overset{3}{\cancel{21}} \times 15 \times \overset{2}{\cancel{18}}}{\underset{1}{\cancel{7}} \times \underset{2}{\cancel{54}} \times \underset{1}{\cancel{7}} \times \cancel{3} \times \cancel{9}} = 60.$

9. $\dfrac{\overset{2}{\cancel{56}} \times 11}{\cancel{28}} = 22.$

10. $\dfrac{\overset{2}{\cancel{14}} \times \overset{8}{\cancel{24}} \times \overset{1}{\cancel{9}}}{\underset{7}{\cancel{63}} \times \underset{1}{\cancel{3}}} = 16 \text{ cents.}$

11. $\dfrac{5 \times \overset{3}{\cancel{12}} \times 7}{\cancel{3} \times \cancel{4}} = 35.$

12. $\dfrac{\cancel{3} \times \overset{2}{\cancel{60}} \times \overset{\overset{5}{\cancel{15}}}{\cancel{75}}}{\underset{3}{\cancel{90}} \times \underset{5}{\cancel{40}}} = 10.$

(Art. 87, p. 57.)

(2.)	(3.)	(4.)
63013 × 17	79245 × 19	32067812 × 16
441091	713205	192406872
1071221	1505655	513084992

(Art. 88, p. 57.)

(2.)	(3.)	(4.)
6651 × 108	111223 × 104	2042 × 1009
53208	444892	18378
718308	11567192	2060378

(ART. **89**, p. **58.**)

(2.)	(3.)	(4.)
13317 × 51	71389 × 21	12062 × 91
66585	142778	108558
679167	1499169	1097642

(ART. **90**, p. **58.**)

(2.)	(3.)	(4.)
8360 × 7001	10613 × 801	91603 × 2001
58520	84904	183206
58528360	8501013	183297603

(ART. **91**, p. **59.**)

(2.)	(3.)	(4.)
915	1224½	180
22⅗	18	69⅙
1830	9792	1620
1830	1224	1080
20130 = product by 22.	22032	30 = ⅙ of 180.
366 = product by ⅗.	2⁴⁄₇ = ⅐ of 18.	12450
20496 = product by 22⅗.	22034⁴⁄₇	

(ART. **92**, p. **59.**)

(2.)	(3.)	(4.)
8)6805600	4)179240	6)19237800
850700	44810	3206300

(5.)	(6.)	(7.)
8)12345678000	3)3130	3)53400
1543209750	1043⅓	17800

(8.)	(9.)
4)771000	6)9168000
192750	1528000

(10.)	(11.)
8)1993000	3)28044000
249125	9348000

(ART. **93**, p. **61**.)

(3.) 61370913
      96488
   490967304 = the product by 8.
  2945803824 = the last product × 6.
 5891607648 = the last product × 2.
5921556653544

(4.)   8649347864 multiplicand.
    1325769612 multiplier.
  103792174368 = the product by 12.
  880337394944 = the foregoing product × 8 for 96.
 4982024369664 = the last product × 6 for 576.
1141713918048 = the first product × 11 for 132.
11467042561708308768 product, Ans.

(ART. **94**, p. **61**.)

(2.)	(3.)
7777777	416231
7777777	416231
77769992223	41622683769

(4.)	(5.)
987654	876543
987654	876543
3)987653012346	3)87653423457
329217670782	29217807819
	2
	58435615638

(6.)	(7.)
999999	32567895
999999	32567895
9998990001	3)325646382105
	108548794035

(8.)
66666
   66666
3)6666533334
  2222177778
        2
  4444355556

(9.)
912345678
  912345678
90322222122

(10.)
1234567
  1234567
12344485433

(11.)
98123452
    98123452
98123353876548

CONTRACTIONS IN DIVISION.

(Art. 95, p. 62.)

(2.)
89630
  3
26889|0

(3.)
123450
  6
7407|00

(4.)
18621
  8
1489|68

(5.)
317121
  4
126848|4

(6.)
876735
  3
26302|05

(7.)
123456
  8
987|648

(8.)
61678500
  4
246714|000

(9.)
9500
  6
57|000

(10.)
12000
  3
36|000

(11.)
150000
  8
1200|000

(12.)
$333\frac{1}{3}$
  3
10|00

(13.)
120
  4
48|0

(14.)
616350
  4
24654|00

(ART. **96,** p. **64.**)

(4.)		(5.)		(5.)	
12332	655	98755	1235	98755	1235
12	987	9	9990	9	9990
1	999			1	9999
12345 Ans.		98765 9999. Ans.		98765 Ans.	

(6.)

	9123456779	876543211
	9	999999990
	1	999999999

9123456789 Ans.

(ART. **97,** p. **64.**)

(2.)	(3.)
44)39006(886$\frac{22}{44}$	34)1088(32
380	68
286	0
22	

(4.)	(5.)
191)5157(27	7750)1133123(146$\frac{1623}{7750}$.
1337	35812
0	48123
	1623

(ART. **98,** p. **65.**)

(2.)	(3.)
9$\frac{1}{4}$)13120	36) 76672$\frac{5}{7}$
4 4	7 7
37)52480(1418$\frac{14}{37}$	252)536710(2129$\frac{202}{252}$.
37	504
154	327
148	252
68	751
37	504
310	2470
296	2268
14	202

2

(4.)

205)2090⅜

  8   3

615)6271(10⅕⅞⅕

615

121

(5.)

16½)10626

  2   2

33) 21252(644

198

145

132

132

132

(6.)

69⅛)12450

  6   6

415) 74700(180

415

3320

3320

0

(7.)

17¾)5591¼

  4   4

71)22365(315

213

106

71

355

355

(8.)

10½)2667

  2   2

21) 5334(254

42

113

105

84

84

(9.)

272¼)136125

  4   4

1089) 544500(500

5445

00

(10.)

2¾)119

  7   7

17) 833(49

68

153

153

(11.)

31¼)12968¾

  4   4

125) 51875(415

500

187

125

625

625

PROBLEMS.

1. (ART. **99,** p. **67.**) $2763 + $4650 + $ 8950 = $ 16363.
2. $929 — $279 = $ 650.
3. 6476 — 242 = 6234 feet.

4. $1519 + 328 = 1847.$

5. $1963 - 199 = 1764$; $1764 \div 2 = 882$, miles B travelled; $882 + 199 = 1081$, miles A travelled.

6. $\$250 + 410 = \$660$; $\$4698 - 660 = \$4038$; $\$4038 \div 3 = \$1346$, George received; $\$1346 + \$250 = \$1596$, James received; $\$1346 + \$410 = \$1756$, Edwin received.

7. $\$8463 \div 217 = \$39.$

8. $19 \times 3 = 57$; $684 \div 57 = 12$ weeks.

9. $\$3808 \div 224 = 17$ men.

10. $\$575 \times 99 = \$56925.$

11. $96 \times 22 = 2112$; $63360 \div 2112 = 30.$

12. $1101 \times 13 = 14313.$

MISCELLANEOUS EXAMPLES.
(Page **68.**)

1. $200 + 305 + 230 + 282 + 171 = 1188$, Ans.

2. $\$175 + \$87 + \$31 = \$293$; $\$38 + \$12 = \$50$; $\$293 - \$50 = \$243$, Ans.

3. $97 \times 5 = \$485$; $97 - 17 = 80$; $80 \times 8 = \$640$; $\$640 - \$485 = \$155$, gain, Ans.

4. $3787 \times 1728 = 6543936$ cubic inches, Ans.

5. 175686lb. $\div 987 = 178$lb., Ans.

6. $120 + 80 + 160 = 360$ acres, Ans.

7. $\$8395 \div 365 = \$23$, Ans.

8. $12 \times 6 = 72$; $12 \times 12 \times 6 = 864$; $864 - 72 = 792$, Ans.

9. $\$7 \times 8 = \$56$; $\$8 \times 3 = \$24$; $\$56 + \$24 = \$80$, Ans.

10. $\$31 + \$45 = \$76$; $3952 \div 76 = 52$, Ans.

11. $13 \times 4 = 52$, Ans.

12. $\$250,000 \div 500 = \$500$, Ans.

13. $127 + 212 = 339$; $500 - 339 = 161$; $\$47 \times 127 = \$5969$; $\$96 \times 212 = \$20352$; $\$37 \times 161 = \$5957$; $\$5969 + \$20352 + \$5957 = \$32,278$; $\$32,278 - \$17,876 = \$14,402$, Ans.

14. $17 - 7 = 10$; $18 - 10 = 8$ miles, Ans.

15. $15 \times 5 = 75$ days, Ans.

16. $2 + 8 = 10$; $10 \times 4 = 40$; $40 + 32 = 72$; $72 \div 2 = 36$; $36 \times 10 = 360$; $360 \div 24 = 15$, Ans.

17. $16 + 4 = 20$; $20 \times 2 = 40$; $126 + 40 = 166$; $48 \div 2 = 24$; $34 \times 6 = 204$; $17 - 5 = 12$; $204 \div 12 = 17$; $17 + 24 = 41$; $166 - 41 = 125$, Ans.

18. $683 - 16 = 667$; $667 \div 23 = 29$ pupils, Ans.

19. $12 + 40 = 52$; $40 \times 5 = 200$; $12 \times 6 = 72$; $52 + 200 + 72 + 7 = 331$ miles, Ans.

20. $9891 - 1211 = 8680$; $8680 \div 2 = 4340$, A received; $9891 - 4340 = 5551$, B received, Ans.

21. $15 \times 16 = 240$; $4080 \div 240 = 17$, third number, Ans.

22. $\$4 \times 17 = \$68$; $\$2 \times 32 = \$64$; $\$68 + \$64 = \$132$; $132 \div 6 = 22$ tons, Ans.

23. $100 + 200 + 300 = 600$; $5608 - 600 = 5008$; $5008 \div 4 = 1252$, first year; $1252 + 100 = 1352$, second year; $1352 + 100 = 1452$, third year; $1452 + 100 = 1552$, fourth year, Ans.

24. $\$45 - \$35 = \$10$; $\$10 \times 12 = \$120$; $\$1100 - \$620 = \$480$; $480 \div 120 = 4$ years, Ans.

25. $19782 + 31 = 19813$, Ans.

26. $1885 \div 65 = 29$; $\$4473 + \$812 = \$5285$; $\$65 + \$6 = \$71$; $4473 \div 71 = 63$ shares; $63 - 29 = 34$ shares remaining; $\$5285 - \$1885 = \$3400$; $\$3400 \div 34 = \$100$, Ans.

UNITED STATES MONEY.

(Art. 107, p. 73.)		
2.	76500 cents.	
3.	$72\frac{6}{10}$ cents.	
4.	$\$3.29$	
5.	$\$12.345$	
6.	123560 mills.	
7.	2220 cents.	

ADDITION.

	(ART. 108, p. 73.)	6.	$1717.062
3.	$328.722	7.	$154.355
4.	$1805.847	8.	$7.105
5.	$3320.675		

SUBTRACTION.

	(ART. 109, p. 74.)	7.	$13.875
3.	$278.486	8.	$909.75
4.	$69.066	9.	$3.43
5.	$154.069	10.	$2.36
6.	$2058.08		

MULTIPLICATION OF UNITED STATES MONEY.

	(ART. 110, p. 75.)	10.	$44.748
2.	$16.38	11.	$109.25
3.	$58.59	12.	$790.92
4.	$591.25	13.	$65.10
5.	$3.358	14.	$142.02
6.	$249.28	15.	$48.222
7.	$1120.32	16.	$48910.95
8.	$358.28	17.	$67.16
9.	$102.69	18.	$70.664

DIVISION OF UNITED STATES MONEY.

	(ART. 111, p. 77.)	13.	$3.75
4.	$1.625	14.	$2.28
5.	425	15.	47
6.	$3.89	16.	17
7.	$1.75	17.	691
8.	329	18.	$4.68
9.	$0.75	19.	$0.18
10.	$4.00	20.	$132.55
11.	144	21.	$1.12
12.	$0.06		

2*

PRACTICAL QUESTIONS BY ANALYSIS.

	(Art. **116,** p. **79.**)		
		27.	$60.17½
2.	$118.82	28.	$12467.25
3.	$7.24½	30.	$28.20
4.	$310.20	31.	$42.75
5.	$1126.93½	32.	$9.035¼
6.	$4668595.00	33.	$771.65
8.	$120.66⅔	34.	$283.12½
9.	$105.00	35.	$35.526⅞
10.	$72.68¾	37.	$0.13
12.	$3800.00	38.	$4.16⅔
13.	$337.50	39.	$72.25
14.	$502.25	40.	$29.70
15.	$22.50	41.	$2.50
16.	$6996.00	42.	$0.22
18.	$190.00	43.	$0.20
19.	$3255.75	45.	152
20.	$103.35	46.	84
21.	$52.95	47.	362
22.	$129.525	48.	27
24.	$876.875	49.	50
25.	$472.50	50.	216
26.	$7.98¾		

BILLS.

(Art. **122,** p. **83.**)

(1.) JAMES DOW.		(2.) SAMUEL SMITH.	
$0.45 × 17 =	$7.65	$0.98 × 13 =	$12.74
.37 × 19 =	7.03	.15 × 16 =	2.40
.46 × 16 =	7.36	.13 × 36 =	4.68
.87 × 13 =	11.31	.9 × 47 =	4.23
.63 × 9 =	5.67	.19 × 12 =	2.28
.56 × 25 =	14.00	.17 × 7 =	1.19
.31 × 17 =	5.27	.61 × 13 =	7.93
.16 × 19 =	3.04		
	$61.33		$35.45

(3.) WILSON, NILES & Co.

$6.00 × 2 = $12.00
5.00 × 3 = 15.00
1.80 × 5 = 9.00
.25 × 17 = 4.25
.60 × 13 = 7.80
.50 × 19 = 9.50
.60 × 3 = 1.80
1.12½ × 7 = 7.87½
.22 × 15 = 3.30
15.00 × 5 = 75.00
4.50 × 3 = 13.50
1.50 × 5 = 7.50
4.50 × 1 = 4.50

$171.02½

(4.) ALBERT CRAWFORD Dr.

$5.25 × 17 = $89.25
1.62 × 29 = 46.98
.17 × 60 = 10.20
.27 × 49 = 13.23
3.19 × 18 = 57.42
2.75 × 27 = 74.25
.61 × 75 = 45.75
.75 × 36 = 27.00
.18 × 49 = 8.82

$372.90

Cr.

Cash, $83.00
$30.00 × 3 = 90.00
4.00 × 7 = 28.00
2.00 × 4 = 8.00
1.75 × 5 = 8.75
2.25 × 7 = 15.75
Cash,   = 60.00
Draft,   = 45.00

$338.50

Remains due, $34.40

(5.) BENJAMIN TREAT.

$25.50 × 37 = $943.50
16.17 × 41 = 662.97
97.75 × 40 = 3910.00
169.37 × 13 = 2201.81

$7718.28

(6.) J. C. PORTER.

$8.25 × 17 = $140.25
.50 × 50 = 25.00
.08½ × 140 = 11.90
.63 × 120 = 75.60

$252.75

(7.)   JOHN CUMMINGS.

$6.25 × 97 =	$606.25	Am't brought up $12811.36
5.95 × 167 =	993.65	1.61 × 39 = 62.79
6.07 × 87 =	528.09	.17 × 197 = 33.49
5.75 × 196 =	1127.00	.69 × 86 = 59.34
7.25 × 275 =	1993.75	1.17 × 78 = 91.26
1.16 × 69 =	80.04	.85 × 187 = 158.95
.67 × 136 =	91.12	11.61 × 91 = 1056.51
.76 × 68 =	51.68	17.15 × 83 = 1423.45
1.37 × 169 =	231.53	3.16 × 47 = 148.52
9.67 × 76 =	734.92	18.15 × 35 = 635.25
69.70 × 89 =	6203.30	9.47 × 47 = 445.09
3.47 × 49 =	170.03	6.83 × 57 = 389.31
Am't carried up, $12811.36		$17315.32

LEDGER ACCOUNTS.

(ART. 123, p. 87.)

1. $ 461.97   2. $ 3165.60
3. $ 4130.23   4. $ 21995.54

REDUCTION OF COMPOUND NUMBERS.

ENGLISH MONEY.

(ART. 128, p. 90.)

(3.)   (4.)
127£. 15s. 8d.   4)122672far.
20   12)30668d.
─────   ───────
2555s.   20)2555s. 8d.
12   ─────────
─────   127£. 15s. 8d.
30668d.
4
─────
122672far.

(5.)

28£. 19s. 11d. 3far.
20
———
579s.
12
———
6959d.
4
———
27839far.

(6.)

4)27839far.
12)6959d. 3far.
20)579s. 11d.
28£. 19s. 11d. 3far.

(7.)

378£.
20
———
7560s.
12
———
90720d.

(8.)

12)90720d.
20)7560s.
378£.

AVOIRDUPOIS WEIGHT.

(Art. 129, p. 91.)

(1.) 165T. 13cwt. 3qr. 19lb. 14oz.
20
———
3313cwt.
4
———
13255qr.
25
———
66284
26511
———
331394lb.
16
———
1988368
331395
———
5302318oz.

(2.) 16)5302318oz.
25)331394lb. 14oz.
4)13255qr. 19lb.
20)3313cwt. 3qr.
165T. 13cwt. 3qr. 19lb. 14oz.

(3.) 3T. 16cwt. 2qr. 18lb.
```
      20
     ────
     76cwt.                (4.) 16)122688oz.
      4                         25)7668lb.
     ────                         4)306qr. 18lb.
     306qr.                      20)76cwt. 2qr.
      25                          3T. 16cwt. 2qr. 18lb.
     ────
     1538
     613
     ────
     7668lb.
      16
     ────
     46008
     7668
     ────
     122688oz.
```

(5.) 2T. 17cwt. 3qr. 16lb. 15oz. 13dr.
```
      20
     ────
     57cwt.       (6.) 16)1482749dr.
      4                16)92671oz. 13dr.
     ────              25)5791lb. 15oz.
     231qr.             4)231qr. 16lb.
      25              20)57cwt. 3qr.
     ────             Ans. 2T. 17cwt. 3qr. 16lb. 15oz. 13dr.
     1161
     463
     ────
     5791lb.
      16
     ────
     34751                (7.) 7T. 17cwt.      (8.) 19cwt. 3qr. 20lb.
     5792                       20                    4
     ────                      ────                  ────
     92671oz.                  157cwt.                79qr.
      16                        4                     25
     ────                      ────                  ────
     556029                    628qr.                 395
     92672                      25                    160
     ──────                    ────                  ────
     1482749dr., Ans.          3140                  1995lb.
                               1256                   .09
                               ────                  ────
                               15700lb.              $179.55, Ans.
                                .07
                               ────
                               $1099.00, Ans.
```

TROY WEIGHT.

(Art. **130**, p. **92**.)

(1.) 28lb. 11oz. 12pwt. 15gr.
　　　　　12
　　　　——
　　　　347oz.
　　　　　20
　　　　——
　　　6952pwt.
　　　　24
　　——
　　27813
　13905
　——
166863gr., Ans.

(2.) 24)166863gr.
　　　　20)6952pwt. 15gr.
　　　　　12)347oz. 12pwt.
　　　　Ans. 28lb. 11oz. 12pwt. 15gr.

(3.) 3lb. 10oz.
　　　　12
　　　——
　　　46oz.
　　　20
　　　——
　　920pwt.
　　　24
　——
22080gr., Ans.

(4.) 24)22080gr.
　　　　20)920pwt.
　　　　12)46oz.
　　　　　3lb. 10oz., Ans.

(5) 73lb. 11oz.
　　　　12
　　　——
　　887oz.
　　　20
　　——
17740pwt.
　$.062
　——
　35480
106440
——
$1099.88, Ans.

(6.) .062)$1099.88
　　　　20)17740pwt.
　　　　12)887oz.
　　　Ans. 73lb. 11oz.

(7.) 57lb. 7oz.	(8.) 19lb. 6oz. 16pwt.	(9.) 13lb. 9oz.
12	12	12
691oz.	234oz.	165
20.593½	20	1.3857
2073	4696pwt.	1155
6219	.93	825
3455	$4367.28, Ans.	1320
1382		495
138½		165
$14229.901½, Ans.		$228.6405, Ans.

APOTHECARIES' WEIGHT.

(Art. 131, p. 93.)

(1.) 23℔ 9℥ 0ʒ 2℈ 13gr.
   12
   285℥
    8
   2280ʒ
    3
   6842℈
    20
   136853gr., Ans.

(2.) 20)136853gr.
     3)6842℈
     8)2280ʒ
     12)285℥
     Ans. 23℔ 9℥ 0ʒ 2℈ 13gr.

(3.) 23℔
     12
    276℥
     8
    2208ʒ
     3
    6624℈, Ans.

(4.) 3)6624℈
     8)2208ʒ
     12)276℥
        23℔, Ans.

(5.) 47℔ 0℥ 0ℨ 1Ɖ 19gr.   (7.) 1℔

    12   12

  564ℨ   (6.) 20)270759gr.   12℥

    8   3)13537Ɖ 19gr.   8

 4512ℨ   8)45123 1Ɖ   96ℨ

    3   12)564ℨ   3

13537Ɖ   Ans. 47℔ 0℥ 0ℨ   288Ɖ

  20   [1Ɖ 19gr.   20

270759gr., Ans.   5)5760gr.

                                 1152

                                 12½

                            $144.00

                             1.80

                       $142.20, Ans.

AVOIRDUPOIS, TROY, AND APOTHECARIES' WEIGHT COMPARED.

(Art. 132, p. 94.)

(1.) 13lb. 6oz.   (2.) 16lb. 3oz. 1pwt. 1gr.

   7000   437½gr.   12

 91000   6   195oz.

  2625 2625gr.   20

5760)93625(16lb.   3901pwt.

  576   24

 3602   7000)93625(13lb.

 3456   7000

 1465   23625

   12   21000

5760)17580(3oz.   2625

 1728   16

  300   7000)42000(6oz.

   20   42000

6000 (Carried forward to p. 26.)

        3

(Brought forward.)
5760)6000(1pwt.
    5760
    ────
     240
      24
    ────
5760)5760(1gr.
     5760

(3.)

3lb. 8oz. 10pwt.
  12
 ────
 44oz.
  20
 ────
 890pwt.
  24
 ─────
20)21360gr.
 3)1068
 ──────
 356dr., Ans.

(4.)

356dr.
   3
 ────
1068
  20
─────
24)21360
20)890
12)44oz. 10pwt.
Ans. 3lb. 8oz. 10pwt.

(5.)

2lb. 8oz.   2lb. 8oz.   437½gr.
  16   7000   8
 ────   ─────   ──────
 40oz.   14000   3500gr.
 2.40   3500
────   ──────
$96.00   20)17500gr.
              875Ɵ
               .20
             ─────
             $175
               96
             ─────
             $ 79 gain, Ans.

(6.)

12lb.   12lb.
  16   7000
 ────   ──────
 192oz.   24)84000gr.
  37½   20)3500pwt.
 ────   175oz.
$ 72.00   .40
   70   ──────
────── $ 70.00
Ans. $ 2.00 loss.

LINEAR OR LONG MEASURE.

(ART. 133, p. 96.)

(1.) 96deg. 56m. 7fur. 32rd. 12ft. 6in.
　　　69⅙
　　　────
　　　870　　　　(2.) 12)424320486in.
　　　581　　　　　　16½)35360040ft. 6in.
　　　 16　　　　　　　40)2143032rd. 12ft.
　　6696m.　　　　　　8)53575fur. 32rd.
　　　 8　　　　　　69⅙)6696m. 7fur.
　53575fur.　　　　　Ans. 96deg. 56m. 7fur. 32rd.
　　 40　　　　　　　　[12ft. 6in., Ans.
　2143032rd.
　　 16½
　12858194　　　(3.) 79m.　　(4.) 16½)417120ft.
　2143033　　　　　 8　　　　40)25280rd.
　1071516　　　　632fur.　　　　8)632fur.
　35360040ft.　　　 40　　　　Ans. 79m.
　　 12　　　　25280rd.
Ans. 424320486in.　　16½
　　　　　　　　417120ft., Ans.

(5.) 396　　　　　　(6.) 12)3136320in.
　　 40　　　　　　　　16½)261360ft.
　15840rd.　　　　　　2　　　2
　　 16½　　　　　　 ────────
　95040　　　　　　　33)522720
　15840　　　　　　　　40)15840rd.
　 7920　　　　　　　　　396fur., Ans.
　261360ft.
　　 12
　3136320in., Ans.

(7.)   30ms.
     8
   ————
   240fur.
     40
   ————
   9600rd.
    16½
   ————
   57600
   9600
   4800
   ————
   158400ft.
     12
  ——————
  1900800in., Ans.

(8.)   12)1900800in.
     16½)158400ft.
     2   2
    ——————
    33)316800
     40)9600rd.
     8)240fur.
      30ms., Ans.

CLOTH MEASURE.

(ART. 134, p. 96.)

(1.)
17yd. 3qr. 2na.
   4
  ————
  71qr.
   4
  ————
  286na., Ans.

(2.)
4)286na.
4)71qr. 2na.
17yd. 3qr. 2na., Ans.

(3.)
365yd. 1qr. 3na.
    4
  ————
  1461qr.
    4
  ————
  5847na., Ans.

(4.)
4)5847na.
4)1461qr. 3na.
  365yd. 1qr. 3na., Ans.

(5.)
71E.E. 4qr.
  5
  ————
  359qr.
   4
  ————
  1436na., Ans.

(6.)
4)1436na.
5)359qr.
Ans. 71E.E. 4qr.

(7.)

47yd. 3qr.
    4
—————
191qr.
1.25
—————
955
382
191
—————
$ 238.75, Ans.

(8.)

31¼E.E.
  5
—————
4)156qr.
—————
39
3
—————
$117, Ans.

SURFACE OR SQUARE MEASURE.

(Art. 135, p. 97.)

(1.)   57A. 3R. 27p. 21yd. 8ft. 57in.
     4
—————
231R
  40
—————
9267p.
  30¼
—————
278031
 2316¾
—————
280347½yds.
    9
—————
2523137¾ft.
  144
—————
10092555
10092553
2523127
  108
—————
363331893in., Ans.

(2.)

144)363331893in.
—————
9)2523138ft. 21in.
—————
30¼)280348yd. 6ft.
4   4
—————
121)1121392
—————
40)9267p. 85 ÷ 4 = 21yd. 2ft. 36in.
4)231R. 27p.
—————
57A. 3R. 27p. 21yd. 2ft. 36in.
      6   21
—————
Ans. 57A. 3R. 27p. 21yd. 8ft. 57in.

(3.)   25A.

     4

  100R.

    40

 4000p.

  272¼

  8000

28000

 8000

 1000

1089000ft., Ans.

(4.) 272¼)1089000ft.

    4   4

1089)4356000

      40)4000p.

       4)100R.

       25A., Ans.

(5.) 365S. **M.**

     640

  14600

  2190

233600A.

   160

14016000

  2336

Ans. 37376000sq. rd.

(6.) 160)37376000 square rods.

    640)233600 acres.

       365 square miles.

(7.) 144)12345678

     9)85733ft. 126 in.

   30¼)9525yd. 8ft.

    40)314p. 26½yd.

     4)7R. 34p.

       1A. 3R.

1A. 3R. 34p. 26½yd. 8ft. 126in.

       ½yd. = 4ft. 72in.

Ans. 1A. 3R. 34p. 27yd. 4ft. 54in.

(8.) 39A. 2R. 16p.

     4

  158R.

   40

 6336p.

 3.75

31680

44352

19008

$23760.00

  6336 poles.

   272¼

 12672

44352

12672

 1584

1724976 square feet.

  0.25

8224880

3449952

431244.00

23760.00

$407,484.00, Ans.

SURVEYOR'S MEASURE.

(1.)   (Art. **136,** p. **98.**)   (2.)

```
46m. 3fur. 5ch. 25l.       100)3715251.
     8                        10)3715ch. 25l.
  ─────                          8)371fur. 5ch.
  371fur.                          46m. 3fur.
     10                   Ans. 46m. 3fur. 5ch. 25l.
  ─────
  3715ch.
    100
  ─────────
  3715251., Ans.
```

(3.)   (5.)   (6.)

```
97m.        1377sq. ch.      $80 ÷ 160 = $0.50.
  8            16         625)2,126,250 square links.
─────        ─────           ─────────────
776fur.      8262               3402sq. rd.
  10         1377                  0.50
─────      40)22032              ─────────
7760ch.      ──────          $1701.00, Ans.
  100      4)550R. 32p.
─────        ─────────
7760001., Ans.  137A. 2R.
             137A. 2R. 32p., Ans.
```

CUBIC OR SOLID MEASURE.

(1.)   (Art. **137,** p. **100.**)   (2.)

```
   29 cords.        1728)6414336 cubic inches.
  128                  128)3712 cubic feet.
 ─────                     29 cords, Ans.
  232      (3.)
   58      19 tons.
   29        40                (4.)
 ─────      ─────        1728)1313280 cubic inches.
 3712ft.    760ft.          40)760 feet.
 1728       1728               19 tons, Ans.
 ─────      ─────
 29696      6080
  7424      1520
 25984      5320
  3712       760
─────       ─────
Ans. 6414336in.   1313280 cubic inches, Ans.
```

(5.) 128 × 128 = 16384 cubic feet, Ans.

(6.) 16384 ÷ 128 = 128 cords, Ans.

(7.) 4 × 40 × 7 = 1120 cubic feet, Ans.

(8.) 8650ft. ÷ 128 = 67 cords, 74 cubic feet, Ans.

(9.) 17 × 11 × 9 = 1683 cubic feet, Ans.

LIQUID OR WINE MEASURE.

(ART. 138, p. 101.)

(1.) 57T. 3hhd. 50gal. 3qt.

$$\frac{4}{231\text{hhd.}}$$

$$\frac{63}{14603\text{gal.}}$$

$$\frac{4}{58415\text{qt.}}$$

$$\frac{2}{116830\text{pt.}}$$

(2.)

2)116830 pints.

4)58415qt.

63)14603gal. 3qt.

4)231hhd. 50gal.

57T. 3hhd.

57T. 3hhd. 50gal. 3qt., Ans.

(3.) 96hhd. 47gal. 2qt.

$$\frac{63}{}$$

295

580

$$\overline{6095\text{gal.}}$$

$$\frac{4}{24382\text{qt.}}$$

$$\frac{2}{48764\text{pt.}}$$

$$\frac{4}{195056 \text{ gills, Ans.}}$$

(4.)

4)195056 gills.

2)48764pt.

4)24382qt.

63)6095gal. 2qt.

96hhd. 47gal.

Ans. 96hhd. 47gal. 2qt.

(5.)

40hhd.

$$\frac{63}{}$$

120

240

$$\overline{2520\text{gal.}}$$

$$\frac{4}{10080\text{qt.}}$$

$$\frac{2}{20160\text{pt.}}$$

0.37½

141120

60480

10080

$$\overline{\$7560.00, \text{ Ans.}}$$

(6.) 2

$$\frac{63}{126}$$

0.40

$$\overline{\$50.40}$$

126

$$\frac{4}{504}$$

0.12

$$\overline{\$60.48}$$

50.40

$$\overline{\$10.08, \text{ Ans.}}$$

BEER MEASURE.

(Art. 139, p. 102.)

(1.) $46 \times 54 = 2484$gal.; 2484gal. $+ 49$gal. $= 2533$gal.; $2533 \times 4 = 10132$qt.; $10132 \times 2 = 20264$pt., Ans.

(2.) $20264 \div 2 = 10132$qt.; $10132 \div 4 = 2533$gal.; $2533 \div 54 = 46$hhd. 49gal., Ans.

(3.) $368 \times 54 = 19872$gal.; $19872 \times 4 = 79488$qt.; $79488 \times 2 = 158976$pt., Ans.

(4.) $158976 \div 2 = 79488$qt.; $79488 \div 4 = 19872$gal.; $19872 \div 54 = 368$hhd., Ans.

(5.) $76 \times 54 = 4104$gal.; $4104 \times 0.29 = \$1190.16$, Ans.

(6.) $47 \times 54 = 2538$gal.; 2538gal. $+ 36$gal. $= 2574$gal.; $2574 \times 4 = 10296$qt.; $10296 \times 0.05 = \$514.80$, Ans.

DRY MEASURE.

(Art. 140, p. 103.)

(1.) $35 \times 4 = 140$pk.; 140pk. $+ 3$pk. $= 143$pk.; $143 \times 8 = 1144$qt.; $1144 \times 2 = 2288$pt., Ans.

(2.) $2288 \div 2 = 1144$qt.; $1144 \div 8 = 143$pk.; $143 \div 4 = 35$bu. 3pk., Ans.

(3.) $676 \times 36 = 24336$bu.; $24336 \times 4 = 97344$pk., Ans.

(4.) $97344 \div 4 = 24336$bu.; $24336 \div 36 = 676$ch., Ans.

(5.) $50 \times 0.60 = \$30.00$; $50 \times 5 = 250$pk.; $250 \times 0.15 = \$37.50$; $\$37.50 - \$30.00 = \$7.50$ gain, Ans.

(6.) $96 \times 0.42 = \$40.32$; $96 \times 24 = 2304$lb.; $2304 \div 32 = 72$bu.; $72 \times 0.45 = \$32.40$; $\$40.32 - \$32.40 = \$7.92$ loss, Ans.

DRY, LIQUID, AND BEER MEASURE COMPARED.

(Art. **141,** p. **104.**)

(1.) 4hhd. 15gal. beer = 231gal.; $231 \times 282 = 65142$; $65142 \div 231 = 282$gal. wine = 4hhd. 30gal., Ans.

(2.) 4hhd. 30gal. = 282 wine gal.; $282 \times 231 = 65142$; $65142 \div 282 = 231$ beer gal. = 4hhd. 15gal., Ans.

(3.) $2820 \times 4 = 11280$qt.; $11280 \times 0.04 = \$451.20$; $2820 \times 282 = 795240$; $795240 \div 231 = 3442\frac{4}{6}$gal. wine measure; $3442\frac{4}{6} \times 4 = 13770\frac{3}{9}$qt.; $13770\frac{3}{9} \times 0.06 = \$826.22\frac{3}{6}$; $\$826.22\frac{3}{6} - \$451.20 = \$375.02\frac{3}{6}$, Ans.

(4.) $2538 \times 4 \times 0.04 = \$406.08$; $2538 \times 231 = 586278$; $586278 \div 282 = 2079$; $2079 \times 4 \times 0.06 = \$498.96$; $\$498.96 - \$406.08 = \$92.88$ gain, Ans.

(5.) $385 \times 4.00 = \$1540.00$; 385bu. $\times 268\frac{4}{5} = 103488$; $103488 \div 231 = 448$bu.; $448 \times 32 \times 0.20 = \$2867.20$; $\$2867.20 - \$1540.00 = \$1327.20$ gain, Ans.

(6.) $1000 \times 1.05 = \$1050.00$; 1000bu. $\times 268\frac{4}{5} = 268800$; $268800 \div 231 = 1163\frac{7}{11}$bu.; $1163\frac{7}{11} \times 1.12 = \$1303.27$; $\$1303.27 - \$1050.00 = \$253.27$ gain, Ans.

(7.) $30 \times 1728 = 51840$ cubic inches; $51840 \div 282 = 183\frac{39}{47}$gal.; $51840 \div 268\frac{4}{5} = 192\frac{6}{7}$gal.; $192\frac{6}{7} - 183\frac{39}{47} = 9\frac{9}{329}$gal., Ans.

(8.) $365 \times 4 \times 0.05 = \$73.00$; $365 \times 231 = 84315$; $84315 \div 282 = 299$gal.; $299 \times 4 \times 0.05 = \$59.80$; $\$73.00 - \$59.80 = \$13.20$, Ans.

TIME.
(Art. 142, p. 106.)

(1.) 365da. 5h. 48min. 49sec.
   24
  ——
  1465
   730
  ——
  8765h.
    60
  ——
  525948m.
   60
  ——
31556929sec., Ans.

(2.)
60)31556929sec.
 60)525948m. 49sec.
  24)8765h. 48m.
   365da. 5h.

Ans. 365da. 5h. 48m. 49sec.

(3.) 2348yrs. + 1855yrs. = 4203yrs.; 365da. 6h. = 31557600 sec.; 31557600sec. × 4203 = 132636592800sec., Ans.

(4.) 31556929 seconds in a solar year; therefore 74726807872 ÷ 31556929 = 2368 years, Ans.

(5.) June 13da. + July 31 + August 31 + September 30 + October 31 + November 30 + December 31 + January 31 + February 16 + 365 + 365 = 974da. = 2 yrs. 244da., Ans.

CIRCULAR MEASURE.
(Art. 143, p. 107.)

(1.) 27S. 27° 43′
   30
  ——
  837°
   60
  ——
 50263′, Ans.

(2.)
60)50263′
 30)837° 43′
  27S. 27°

Ans. 27S. 27° 43′.

(3.) 44 × 30 = 1320°; 1320° + 18° = 1338°; 1338 × 60 = 80280′; 80280′ + 57′ = 80337′; 80337 × 60 = 4820220″; 4820220″ + 23″ = 4820243″, Ans.

(4.) 4820243 ÷ 60 = 80337′ 23″; 80337 ÷ 60 = 1338° 57′; 1338 ÷ 30 = 44S. 18°; 44S. 18° 57′ 23″, Ans.

(5.) 360 × 60 = 21600′; 21600 × 60 = 1296000, Ans.

(6.) 1296000 ÷ 60 = 21600′; 21600 ÷ 60 = 360°, Ans.

' MISCELLANEOUS.

(ART. **144,** p. **108.**)

(1.) $4 \times 5 + 4 = 24$; $24 \times 2 + 1 = 49$; $49 \times 20 + 10 = 990$; $990 \times 24 = 23760$ sheets, Ans.

(2.) $23760 \div 24 = 990$qr.; $990 \div 20 = 49$rm. 10qr.; $49 \div 2 = 24$bun. 1rm.; $24 \div 5 = 4$ba. 4bun.; 4ba. 4bun. 1rm. 10qr., Ans.

(3.) $10 \times 8 + 6 = 86$; $86 \times 21\frac{1}{2} + 8 = 1857$; $1857 \times 14 = 25998$lbs.

(4.) $25998 \div 14 = 1857$; $1857 \div 21\frac{1}{2} = 86$ pigs 8 stone; $86 \div 8 = 10$ fother 6p.; 10 fother 6 pigs 8 stone, Ans.

(5.) $56 \times 12 \times 0.23 = \$154.56$, Ans.

(6.) $\$22.00 \div 200 = \$0.11$, Ans.

(7.) $302 \times 8 \times 0.30 = \$724.80$, Ans.

(8.) $\$4.00 \div 100 = \$0.04$; $50.24 \div .04 = 1256$ pounds.

(9.) $12 \times 12 \times 12 \times \$0.045 = \$77.76$.

(10.) $56 \times 2 = 112$lb.; $112 \times 0.20 = \$22.40$; 196lb. $\div 2 = 98$lb.; $98 \times 0.05 = \$4.90$; $\$22.40 - \$4.90 = \$17.50$, Ans.

MISCELLANEOUS EXAMPLES IN REDUCTION.

(PAGE **109.**)

(1.) $57 \times 20 + 15 = 1155$s.; $1155 \times 12 = 13860$d.; $13860 \times 2 = 27720$ half-pence, Ans.

(2.) $59 \times 12 = 708$oz.; $708 \times 20 + 13 = 14173$pwt.; $14173 \times 24 + 15 = 340167$gr., Ans.

(3.) $340167 \div 24 = 14173$pwt. 15 gr.; $14173 \div 20 = 708$oz. 13pwt.; $708 \div 12 = 59$lb.; 59lb. 13pwt. 15gr., Ans.

(4.) $761 \times 4 = 3044$qr.; $3044 \div 5 = 608$E.E. 4qr., Ans.

(5.) $61 \times 3 = 183$qr.; $183 \div 4 = 45$yd. 3qr., Ans.

(6.) $63 \times 4 = 252$qt.; $252 \times 2 = 504$pt.; $504 \div 3 = 168$ bottles, Ans.

(7.) 2ft. 8in. $= 32$in.; $15 \times 8 \times 40 \times 16\frac{1}{2} \times 12 = 950400$in.; $950400 \div 32 = 29700$ times, Ans.

(8.) 2oz. 12pwt. $= 52$pwt.; 5lb. 2oz. 8pwt. $= 1248$pwt.; $1248 \div 52 = 24$ spoons, Ans.

(9.) 14ft. 9in. = 177in.; 436 × 8 × 40 × 16½ × 12 = 276-
24960in.; 27624960 ÷ 177 = 156073$\frac{139}{177}$ times, Ans.

(10.) 123 × 78 × 4 = 38376 ears; 38376 ÷ 8 = 4797qt.;
4797qt. ÷ 8 = 599pk. 5qt.; 599pk. ÷ 4 = 149bu. 3pk.
5qt., Ans.

(11.) 5yd. 2qr. 3na. = 91na.; 182 × 4 × 4 = 2912na.; 2912
÷ 91 = 32 suits, Ans.

(12.) 5pwt. 10gr. = 130gr.; 3lb. 1oz. 2pwt. 2gr. = 17810gr.;
17810 ÷ 130 = 137 rings, Ans.

(13.) 18in. ÷ 3 = 6in.; 6in. × 4 = 24in.; 56ft. × 60 =
3360ft.; 3360 × 144 = 483840in.; 483840 ÷ 24 =
20160 shingles, Ans.

(14.) 56 × 25 × 2 = 2800sq. ft.; 2800 × 6 = 16800 shin-
gles, Ans.

(15.) 22m. 3fur. 17rd. = 7177rd.; 25000 × 320 = 8000000rd.;
8000000 ÷ 7177 = 1114$\frac{6622}{7177}$da., Ans.

(16.) 7lb. 10oz. = 122oz.; 10cwt. 3qr. 16lb. = 1091lb.; 1091lb.
× 16 = 17456oz.; 17456 ÷ 122 = 143$\frac{5}{61}$ weeks, Ans.

(17.) 7 × 63 = 441gal.; 441 × 4 = 1764qt.; 1764 × 0.09
= $ 158.76, Ans.

(18.) 15 × 54 = 810gal.; 810 × 4 = 3240qt.; 3240 × 2 =
6480pt.; 6480 × 0.03 = $ 194.40, Ans.

(19.) 73 × 32 = 2336qt.; 2336 × 0.02 = $ 46.72, Ans.

(20.) 29 × 57 = 1653yd.; 1653 × 0.15 = $ 247.95, Ans.

(21.) 120 × 4½ × 4 = 2160 cubic feet; $ 4.00 ÷ 128 =
$ 0.03125; 2160 × $ 0.03125 = $ 67.50, Ans.

(22.) 33 × 18 = 594sq. ft.; 594 ÷ 9 = 66sq. yd.; $ 0.20 ×
66 = $ 13.20, Ans.

(23.) 20 × 25 = 500 pills; 6 + 5 + 4 = 15gr.; 500 × 15 =
7500gr., Ans.

(24.) 87 × 63 = 5481gal.; 5481 × $ 0.33 = $ 1808.73, Ans.

(25.) 128ft. ÷ 2 = 64ft.; 64 × 880 = 56320oz.; 128 × 480
= 61440oz.; 56320oz. + 61440oz. = 117760oz.; 117-
760 ÷ 16 = 7360lb,, Ans.

4

(26.) $24 \times 0.43 = \$10.32$; $\$1548.00 \div 10.32 = \$150.00$, Ans.

(27.) From 1837 to 1857 there are 5 leap years, viz., 1840, 1844, 1848, 1852, and 1856. Therefore, $\$10.00 \times 5 = \$50.00$; $\$200.00 - \$50.00 = \$150.00$, Ans.

(28.) $1,000,000 \times 25\frac{4}{5} = 25,800,000$gr.; $25,800,000 \div 7000 = 3685\frac{5}{7}$lb., Ans.

(29.) $188 \times 0.50 = \$94.00$; 188gal. $\div 2 = 94$gal.; $94 \times 231 = 21714$; $21714 \div 282 = 77$gal.; $77 \times 4 = 308$qt.; $308 \times 0.12\frac{1}{2} = \$38.50$; $94 \times 4 = 376$qt.; $376 \times 0.14 = \$52.64$; $\$52.64 + \$38.50 = \$91.14$; $\$94.00 - \$91.14 = \$2.86$, Ans.

ADDITION OF COMPOUND NUMBERS.

(Art. 145, p. 112.)

2. 324T. 15cwt. 2qr. 15lb. 12oz. 9dr.
3. 233cwt. 1qr. 6lb. 4oz. 5dr.
4. 209lb. 7oz. 15pwt. 8gr.
5. 385lb. 7oz. 5pwt. 10gr.
6. 183℔ 6℥ 3ʒ 1℈ 19gr.
7. 246℔ 10℥ 3ʒ 2℈ 2gr.
8. 195deg. 55m. 5fur. 24rd. 1ft. 1in.
9. 317m. 3fur. 18rd. 4yd. 2ft. 3in.
10. 229yd. 3qr. 3na. 1¼in.
11. 2513E.E. 3qr. 2na. 0in.
12. 2038A. 1R. 13p. 2ft. 95in.
13. 317A. 0R. 24p. 13¾ft.
14. 160m. 0fur. 1ch. 1p. 12l.
15. 194m. 6fur. 9ch. 0p. 12l.
16. 335T. 23ft. 1173in.
17. 378C. 96ft. 1460in.
18. 354tu. 0hhd. 30gal. 1qt.
19. 286hhd. 42gal. 1qt. 1pt.
20. 784hhd. 24gal. 2qt. 0pt.

21. 298bu. 0pk. 4qt. 1pt.
22. 128bu. 2pk. 7qt.
23. 227y. 7m. 16da. 21h. 28min. 2sec.
24. 240w. 1da. 23h. 33min. 8sec.
25. 5S. 10° 35′ 16″.
26. 11S. 2° 22′ 42″.
27. 20£. 17s. 10d.
28. 5lb. 9oz. 4pwt. 20gr.
29. 7℔ 10℥ 3ℨ 1Ə 4gr.
30. 117cwt. 1qr. 22lb.
31. 175yd. 2qr. 2na.
32. 1060m. 5fur. 16rd. 5yd. 1ft.
33. 588A. 1R. 31p.
34. 7S. 29° 33′ 56″.

SUBTRACTION OF COMPOUND NUMBERS.

(ART. 146, p. 116.)

1. 435£. 16s. 3d.
2. 67£. 18s. 5¾d.
3. 435£. 15s. 9¼d.
4. 51T. 18cwt. 2qr. 21lb. 9oz. 8dr.
5. 53cwt. 3qr. 20lb. 14oz.
6. 54lb. 4oz. 14pwt. 19gr.
7. 38lb. 7oz. 14pwt. 13gr.
8. 52℔ 6℥ 3ℨ 1Ə 15gr.
9. 5℔ 5℥ 0ℨ 1Ə 17gr.
10. 6m. 7fur. 38rd. 2ft. 11in.
11. 9deg. 4m. 5fur. 37rd. 4yd. 1ft. 4in.
12. 48yd. 2qr. 2na. 1¼in.
13. 31E.E. 4qr. 2na.
14. 38A. 2R. 5p. 264ft. 33in.
15. 3A. 1R. 38p. 18yd. 7ft. 36in.
16. 1m. 7fur. 1ch. 2p. 23l.
17. 12m. 6fur. 8ch. 0p. 24l.

18. 30T. 37ft. 1620in.
19. 163C. 53ft. 1289in.
20. 68tu. 1hhd. 53gal. 3qt.
21. 577hhd. 52gal.
22. 17ch. 35bu.
23. 12bu. 3pk. 4qt.
24. 4mo. 23da. 20h. 45min. 42sec.
25. 28y. 1mo. 15da. 21h. 48min. 5sec.
26. 9S. 2° 56′ 52″.
27. 2S. 27° 21′ 54″.
28. 67A. 0rd. 38p. 265¼ft.
29. 99£. 16s. 5½d.
30. 24956m. 7fur. 22rd. 15ft. 9in.
31. 4C. 50ft.
32. 11yd. 1qr. 3na.
33. 33cwt. 2q. 20lb.
34. 54gal. 0qt. 1pt.
35. 6A. 2R. 32p.
36. 12° 11′ 27″.
37. 86bu. 1pk. 0qt. 1pt.
38. 1T. 19ft. 1418in.

MULTIPLICATION OF COMPOUND NUMBERS.

(ART. **147**, p. **119.**)

2. 19£. 10s. 4d.
3. 557T. 19cwt. 1qr. 11lb. 15oz. 7dr.
4. 39lb. 10oz. 13pwt. 3gr.
5. 261lb. 11oz. 0pwt. 0gr.
6. 427℔ 10℥ 0ʒ 2Ә 14gr.
7. 858deg. 44m. 4fur. 6rd. 7ft. 0in.
8. 215m. 7fur. 9ch. 3p. 1l.
9. 181A. 3R. 11p. 6yd. 4ft. 41in.
10. 57yd. 2qr. 3na.
11. 123tu. 8hhd. 36gal. 3qt.
12. 209hhd. 55gal. 3qt. 0pt. 1gi.

13. 118bu. 1pk. 5qt.
14. 110y. 343d. 3h. 41m. 12sec.
15. 149deg. 9m. 5fur. 13rd. 3yd. 1ft.
16. 214A. 3R. 12p.
17. 3d. 16h. 54m.
18. 29bu. 0p. 0qt.

(Art. 148, p. 121.)

2. 56T. 14cwt. 3qr. 15lb.
3. 5£. 4s. 3d.
4. 10£. 8s. 3d.
5. 12£. 13s. 0d.
6. 1290£. 4s. 0d.
7. 32lb. 9oz. 15pwt.
8. 746m. 5fur.
9. 15° 0′ 0″.
10. 1275A. 2R. 16p. 22yd. 8ft. 32in.
11. 2y. 68d. 19h. 54m.

(Art. 149. p. 122.)

2. 89hhd. 52gal. 3qt. 1pt.
3. 179bu. 3pk. 5qt. 0pt. 1gi.
4. 275ch. 19bu. 2pk.
5. 23£. 11s. 5¼d.
6. 3419A. 2R. 23p.
7. 3375yd. 3qr. 1na. 0¼in.
8. 44hhd. 52gal. 2qt. 1pt.
9. 6357m. 5fur. 14rd. 11½ft.
10. 175gal. 2qt. 0pt.
11. 15T. 38ft. 1074in.
12. 132C. 2ft.

4*

DIVISION OF COMPOUND NUMBERS.

(Art. **150,** p. **123.**)

2. 2£. 8s. 9½d.
3. 61T. 19cwt. 3qr. 17lb. 15oz. 15dr.
4. 7lb. 11oz. 14pwt. 15gr.
5. 32lb. 8oz. 17pwt. 12gr.
6. 38℔ 10℈ 5℥ 2℈ 14gr.
7. 71deg. 38m. 2fur. 13rd. 14ft. 4in.
8. 17m. 7fur. 9ch. 3p. 23l.
9. 16A. 2R. 4p. 19yd. 7ft. 79in.
10. 10yd. 3qr. 3na.
11. 17tun. 2hhd. 50gal. 1qt.
12. 29hhd. 61gal. 3qt. 1pt. 3gi.
13. 19bu. 2pk. 7qt. 1pt.
14. 13y. 316d. 15h. 27m. 39sec.
15. 16deg. 39m. 3fur. 39rd. 5yd. 2ft.
16. 23A. 3R. 19½p.
17. 7h. 24m. 30sec.
18. 3bu. 2pk. 4qt.

(Art. **151,** p. **124.**)

2. 12cwt. 2qr. 11lb.
3. 0£. 5s. 9½d.
4. 0£. 9s. 11d.
5. 0£. 11s. 6d.
6. 13£. 8s. 9½d.
7. 3oz. 5pwt. 15gr.
8. 24m. 7fur. 4rd.
9. 0° 15′.
10. 39A. 3R. 17p. 30yd. 8ft. 100in.
11. 2d. 5h. 17m. 19sec.

(Art. **152,** p. **125.**)

2. 2hhd. 19gal 0qt. 1pt.
3. 3bu. 1pk. 4qt. 1pt. 1gi.

   4. 16ch. 7bu. 2pk. 0qt. 0pt.

   5. 0£. 8s. 3¼d.

   6. 117A. 3R. 27p.

   7. 37yd. 3qr. 2na. 2in.

   8. 47gal. 3qt. 1pt.

   9. 17m. 3fur. 13rd. 14ft.

10. 3gal. 1qt. 1pt.

11. 37ft. 978in.

12. 7C. 98ft.

13. 4T. 15cwt. 2qr. 10$\frac{122}{146}$lb.

14. 19m. 3fur. 39rd. 13ft. 2$\frac{7}{11}$in.

15. 24A. 3R. 6$\frac{23}{25}$p.

16. 9A. 1R. 19p. 139$\frac{86}{95}$ft.

17. 13A. 0R. 27p. 3yd. 0ft. 45$\frac{9}{11}$in.

PRINCIPLES AND APPLICATIONS.

(Art. **153**, p. **127**.)

(2.)				(3.)		
y.	mo.	d.		y.	mo.	d.
1857	3	19		1857	5	12
1854	5	3		1856	9	26
2	10	16		0	7	16

(4.)				(5.)		
y.	mo.	d.		y.	mo.	d.
1776	6	4		1799	11	14
1620	11	22		1732	1	22
155	6	12		67	9	22

(Art. **154**, p. **128**.)

2. From Dec. to Feb. = 2 mo.,   31 × 2 = 62 days.

   For Nov. 14d., Feb. 12d.,   16 + 12 = 28

                                         90 days.

3. From July to May = 10mo., $10 \times 31 = 310$ days.
   For June 13 days,   13
                                                          ―――――
                                                               323
   For Sept. 1d., Nov. 1d., Feb. 3d., April 1,   6
                                                          ―――――
                                                               317 days.

4. From April to May = 1mo.,   30 days.
   For March 28 days, May 2 days,   30
                                                        ―――――
                                                           60 days.

5. From Dec. to April = 4mo., $4 \times 31 = 124$ days.
   For Nov. 27 days, April 3 days,   30
                                                         ―――――
                                                            154
   For Feb. 3 days,   3
                                                         ―――――
                                                            151 days.

6. From Feb. to Dec. = 10 mo., $10 \times 31 = 310$ days.
   For Jan. 25 days, Dec. 10 days,   35
                                                         ―――――
                                                            345
   For Feb. 2 days, April 1, June 1, Sept. 1, Nov. 1,   6
                                                   Ans. ―――――
                                                            339 days.

(ART. 155, p. 129.)

2. From April 2 to July 4 = 93 days.
   93 ÷ 7 = 13 weeks, 2 days.
   2 days after Wednesday = Friday, Ans.

3. From Jan. 1 to June 17 = 168 days.
   168 ÷ 7 = 24 weeks, 1 day.
   1 day after Tuesday = Wednesday, Ans.

4. From Jan. 1 to Dec. 25 = 358 days.
   358 ÷ 7 = 51 weeks, 1 day.
   1 day before Tuesday = Monday, Ans.

5. From Nov. 4 to Feb. 1 = 89 days.

   89 ÷ 7 = 12 weeks, 5 days, therefore February will commence 5 days after Tuesday = Sunday. If Sunday be the 1st day, Monday will be the 2nd and 9th days.

6. From Dec. 20 to March 1 = 72 days, it being *leap year*.

   90 − 72 = 18 days; the note will therefore become due March 19. 91 ÷ 7 = 13 weeks. As there is no remainder, and as the note was given on Thursday, it will be payable on Wednesday, March 19.

NOTE. — In reckoning the interest on any note, it is not customary to charge interest on the day the note is paid, but it is reckoned on the day it is given.

—————

DIFFERENCE OF LATITUDE.

(ART. 157, p. 130.)

2. Latitude of Quebec   = 46° 48′ N.
   Latitude of New Orleans   = 29° 57′ N.
   Difference of Latitude   = 16° 51′, Ans.

3. Latitude of Washington City   = 38° 53′ N.
   Latitude of Cape Horn   = 55° 58′ S.
   Difference of Latitude   = 94° 51′, Ans.

4. Latitude of Valparaiso   = 33° 2′ S.
   Latitude of San Francisco   = 37° 48′ N.
   Difference of Latitude   = 70° 50′, Ans.

5. Latitude of New York City   = 40° 42′ N.
   Difference of Latitude N.   = 17° 33′ N.
   Latitude of Havana   = 23° 9′ N., Ans.

6. Latitude of Mobile   = 30° 41′ N.
   Difference of Latitude   = 9° 15′
   Latitude of Philadelphia   = 39° 56′ N., Ans.

DIFFERENCE OF LONGITUDE.

(ART. **159**. p. **131**.)

2. Longitude of Washington City	=	77° 16′ W.
Longitude of Paris	=	2° 20′ E.
Difference of Longitude	=	79° 36′, Ans.
3. Longitude of Cape Flattery	=	124° 43′ W.
Longitude of St. Croix River	=	67° 2′ W.
Difference of Longitude	=	57° 41′, Ans.
4. Longitude of Sacramento City	=	120° 0′ W.
Longitude of Raleigh	=	78° 48′ W.
Difference of Longitude	=	41° 12′, Ans.
5. Longitude of Fort Leavenworth	=	94° 44′ W.
Longitude of Hartford, Ct.	=	72° 40′ W.
Difference of Longitude	=	22° 4′, Ans.
6. Longitude of Honolulu	=	157° 52′ W.
Longitude of Canton	=	113° 14′ E.
		271° 6′

360° — 271° 6′ = 88° 54′, Ans.

LONGITUDE AND TIME.

(ART. **160**, p. **132**.)

2. Longitude of Galveston	=	14° 43′
		4
Difference of time	=	0h. 58m. 52sec.
		12h. 0m. 0sec.
		12h. 58m. 52sec.

3. Longitude of Valparaiso   = 71° 37′ W.
 Longitude of Rome   = 20° 30′ E.
               $\overline{\text{92° 7′}}$
                 4
 Difference of time $= \overline{\text{6h. 8m. 28sec.}}$
              11h. 15m. 0s. A.M.
 Time at Rome $= \overline{\text{5h. 23m. 28s. P.M.}}$

4. Longitude of Jerusalem   = 35° 32′ E.
 Longitude of Baltimore   = 76° 37′ W.
               $\overline{\text{112° 9′}}$
                 4
 Difference of time $= \overline{\text{7h. 28m. 36sec.}}$

 Time at Jerusalem   = 9h. 0m. 0s. A. M.
              7h. 28m. 36s. A. M.
 Time at Baltimore $= \overline{\text{1h. 31m. 24s. A. M., Ans.}}$

(ART. 161, p. 133.)

(2.)   29m. ÷ 4 = 7° 15′
    36sec. ÷ 4 = 0° 9′
        $\overline{\text{7° 24′}}$

(3.) 2h. × 15 = 30°
   20m. ÷ 4 = 5°
   40sec. ÷ 4 = 0° 10′
       $\overline{\text{35° 10′}}$

(4.) 1h. × 15 = 15° 0′
   20sec. ÷ 4 = 0° 5′
       $\overline{\text{15° 5′}}$
       75° 10′
       $\overline{\text{90° 15′ W.}}$

(5.) 53m. ÷ 4 = 13° 15′
   30sec. ÷ 4 = 0° 7′ 30″
       $\overline{\text{13° 22′ 30″}}$

(6.) 1h. × 15 = 15° 0′
   37m. ÷ 4 = 9° 15′
   12sec. ÷ 4 = 0° 3′
      Take $\overline{\text{24° 18′}}$
      From 93° 5′
      $\overline{\text{68° 47′ West.}}$

(7.) 7h. × 15 = 105° 0′
   15m. ÷ 4 = 3° 45′
   40sec. ÷ 4 = 0° 10′
       $\overline{\text{108° 55′}}$
       30° 0′
       $\overline{\text{78° 55′ West.}}$

$$(8.) \quad 4h. \times 15 = 60° \ 0'$$
$$44m. \div 4 = 11° \ 0'$$
$$32sec. \div 4 = \underline{0° \ 8'}$$
$$71° \ 8'$$

MISCELLANEOUS EXAMPLES.

(Page 134.)

(1.) 57 650 000
263 517 496
626 400 000
100 000 000
1 445 000
1 500 000

33)1050 512 496

31,833 712

(2.) 755 — 125 = 630; 630 ÷ 2 = 315; 315 + 48 = 363 bushels in the second; 315 — 48 = 267 bushels in the third.

(3.) 30 × 3 = 90 miles A travels. He has therefore travelled round the island three times. 30 × 5 = 150; 150 ÷ 30 = 5; B has travelled round the island five times. A and B are therefore together.

(4.) $1750 × 2 = $3500; $103 × 19 = $1957; $3500 + $1957 + $113 = $5570, Ans.

(5.) 960 ÷ 2 = 480; 480 ÷ 12 = 40; 40 ÷ 2 = 20 bushels each for those who left; 480 ÷ 8 = 60; 60 ÷ 2 = 30; 30 + 20 = 50 bushels for each who remained.

(6.) 652.00 ÷ .50 = 1304 sheep, Ans.

(7.) 27ft. × 18 = 486sq. ft.; 486 ÷ 9 = 54yd.; 54 × 2.25 = $121.50, Ans.

(8.) 365 × 2½ = 912½h.; 912½h. × 20 = 18250h. in 20 years; 18250 ÷ 12 = 1520da. 10h. = 4y. 60da. 10½h., Ans.

(9.) 7C. 76ft. 1671in. + 16C. 28ft. 56in. + 29C. 127ft. 1000in. + 29C. 10ft. 1216in. = 82C. 115ft. 487in.; 82C. 115ft. 487in. — 45C. 96ft. 0in. = 37C. 19ft. 487in., Ans.

(10.) 42° 21' + 1° 15' = 43° 36'; 43° 36' — 10° 40' = 32° 56', Ans.

(11.) 2000lb. × 2 = 4000lb.; 4000 ÷ 54 = 74$\frac{2}{27}$ cubic feet, Ans.

(13.) $295,300,000 ÷ 250,000 = $1181.20; $295,300,000 ÷ 4 = $73,825,000, Ans.

(14.) $216 \times 1728 = 373248$in.; $373248 \div 231 = 1615\frac{9}{7}\frac{1}{7}$gal.; $1615\frac{9}{7}\frac{1}{7} \div 63 = 25$hhd. $40\frac{9}{7}\frac{1}{7}$gal., Ans.

(15.) Ans. 11th day of May; 131st day of the year.

(16.) 18T. 17cwt. 3qr. 20lb. = 37795lb.; $37795 \times 0.04 =$ $ 1511.80; 4T. 6cwt. 1qr. 14lb. = 8639lb.; $8639 \times$ $0.05 = $ 431.95; 7T. 1cwt. 3qr. 10lb. = 14185lb.; $14185 \times 0.06 = $ 851.10; 8639lb. + 14185lb. = 22824lb.; 37795lb. — 22824lb. = 14971lb.; 14971lb. $\div$ $4 = 3742\frac{3}{4}$lb.; $3742\frac{3}{4} \times 0.06 = $ 224.56\frac{1}{2}; 14971lb. — $3742\frac{3}{4}$lb. = $11228\frac{1}{4}$lb.; $11228\frac{1}{4}$lb. $\div 2 = 5614\frac{1}{8}$lb.; $5614\frac{1}{8} \times 0.10 = $ 561.41\frac{1}{4}; $5614\frac{1}{8} \times 0.12 =$ $ 673.69\frac{1}{2}; $ 673.69\frac{1}{2} \div 2 = $ 336.84\frac{3}{4}; $ 431.95 + $ 851.10 + $ 224.56\frac{1}{2} + $ 561.41\frac{1}{4} + $ 336.84\frac{3}{4} =$ $ 2405.87\frac{1}{2}; $ 2405.87\frac{1}{2} — $ 1511.80 = $ 894.07\frac{1}{2}, Ans.

(17.) 3m. 7fur. 29rd. 15ft. + 5m. 0fur. 10rd. $1\frac{1}{2}$ft. = 9m.; 9m. $\times 6\frac{1}{2} = 58\frac{1}{2}$m., the distance both travel each day; $2691 \div 58\frac{1}{2} = 46$da., the number of days they travel before they meet. 46 days from the 1st of January, omitting 8 Sabbaths, as in duty bound, will be February 23d, 1857, at 2h. 30min. P. M. Swift will have travelled 1186m. 4fur. 22rd. 13ft. 6in.; and Fleet 1504m. 3fur. 17rd. 3ft. As January begins on Thursday, the 23d of February will be on Monday, Ans.

EXAMPLES BY ANALYSIS.
(PAGE 136.)

(1.) $ 8.75 $\div$ 7 = $ 1.25; $ 1.25 $\times$ 20 = $ 25.00, Ans.

(2.) $ 85.00 $\div$ 5 = $ 17.00; $ 17.00 $\times$ 17 = $ 289.00, Ans.

(3.) $ 0.75 $\div$ 3 = $ 0.25; $ 0.25 $\times$ 37 = $ 9.25, Ans.

(4.) $ 1.92 $\div$ 4 = $ 0.48; $ 0.48 $\times$ 37 = $ 17.76, Ans.

(5.) $ 1.08 $\div$ 12 = $ 0.09; $ 0.09 $\times$ 25 = $ 2.25, Ans.

(6.) $ 63.00 $\div$ 9 = $ 7.00; $7.00 $\times$ 27 = $ 189.00, Ans.

(7.) 20m. $\div$ 5 = 4m.; $28 \div 4 = $7h., Ans.

(8.) 49 crackers $\div$ 7 = 7 crackers; 7 crackers $\times$ 19 = 133 crackers, Ans.

(9.) $20 ÷ 5 = $4; 40 × 4 = $160.00, Ans.

(10.) $0.36 ÷ 3 = $0.12; $0.12 × 37 = $4.44, Ans.

(11.) $0.72 ÷ 9 = $0.08; $0.08 × 37 = $2.96, Ans.

(12.) 2£. 17s. 5d. ÷ 52 = 0£. 1s. 1¼d; 0£. 1s. 1¼d. × 76 = 4£. 3s. 11d., Ans.

(13.) 4£. 3s. 11d. ÷ 76 = 0£. 1s. 1¼d.; 0£. 1s. 1¼d. × 52 = 2£. 17s. 5d., Ans.

(14.) 17m. 4fur. 28rd. ÷ 6 = 2m. 7fur. 18rd.; 2m. 7fur. 18rd. × 100 = 293m. 1fur., Ans.

(15.) 41bu. 3pk. 4qt. 1pt. ÷ 7 = 5bu. 3pk. 7qt. 1pt.; 5bu. 3pk. 7qt. 1pt. × 12 = 71bu. 3pk. 2qt.; 71bu. 3pk. 2qt. × 7 = 502bu. 2pk. 6qt., Ans.

(16.) 39A. 2R. 5p. 8yd. 6ft. 108in. ÷ 8 = 4A. 3R. 30p. 20yd.; 4A. 3R. 30p. 20yd. × 9 = 44A. 1R. 35p. 28yd. 6ft. 108in.; 44A. 1R. 35p. 28yd. 6ft. 108in. × 8 = 355A. 3R. 7p. 18yd. 2ft. 36in., Ans.

(17.) 63gal. 3qt. ÷ 4 = 15gal. 3qt. 1pt. 2gi.; 15gal. 3qt. 1pt. 2gi. × 37 = 589gal. 2qt. 1pt. 2gi., Ans.

(18.) 5yd. 3qr. 1na. ÷ 4 = 1yd. 1qr. 3¼na.; 1yd. 1qr. 3¼na. × 36 = 52yd. 1qr. 1na., Ans.

(19.) 11T. 3cwt. 2qr. ÷ 4 = 2T. 15cwt. 3qr. 12lb. 8oz.; 2T. 15cwt. 3qr. 12lb. 8oz. × 23 = 64T. 5cwt. 0qr. 12lb. 8oz., Ans.

(20.) 286da. 4h. 33min. × 12 = 3434da. 6h. 36min., the time one man would dig it; 3434da. 6h. 36min. ÷ 72 = 47da. 16h. 45min. 30sec., time 72 men would dig it, Ans.

(21.) 27yd. 1qr. ÷ 21 = 1yd. 1qr. 0na. 1$in; 1yd. 1qr. 0na. 1$in. × 11 = 14yd. 1qr. 0na. 0$in., Ans.

(22.) 174m. 0fur. 26rd. ÷ 7 = 24m. 6fur. 38rd.; 24m. 6fur. 38rd. × 10 = 248m. 5fur. 20rd., Ans.

(23.) 4 × 60 = 240 pair; $192.00 ÷ 240 = $0.80; $192.00 ÷ 4 = $48.00; $48.00 × 25 = $1200.00, Ans.

(24.) 3A. 2R. 20p. ÷ 4 = 0A. 3R. 25p., cost of one hogshead; 0A. 3R. 25p. × 30 = 27A. 0R. 30p., Ans.

(25.) 20deg. 49m. 5fur. 35rd. 5yd. 0ft. 3in. ÷ 9 = 2deg. 6m. 7fur. 6rd. 4yd. 1ft. 11in.

$$\begin{array}{r}
\text{20deg. 49m.\quad 5fur. 35rd. 5yd.\quad 0ft. 3in.} \\
\underline{10}
\end{array}$$

207deg. 12⅝m. 2fur. 39rd. ½yd. 2ft. 6in.

$$\underline{\text{½yd. = 1ft. 6in.}}$$

207deg. 12⅝m. 2fur. 39rd. 1yd. 1ft. 0in.

$$\underline{\text{⅝m. = 6fur. 26rd. 3yd. 2ft. 0in.}}$$

207deg. 13m. 1fur. 25rd. 5yd. 0ft. 0in., Ans.

FACTORING.

(Page 141.)

(2.)	(3.)	(4.)	(5.)
3)105	2)220	2)936	3)1953
5)35	2)110	2)468	3)651
7)7	11)55	2)234	7)217
1	5)5	3)117	31)31
Ans. 3, 5, 7.	1	3)39	1
	Ans. 2, 5, 11.	13)13	Ans. 3, 7, 31.
		1	
		Ans. 2, 3, 13.	

(6.)	(7.)	(8.)		
2)12462	11)19987	3)225	3)435	2)540
3)6231	23)1817	3)75	5)145	2)270
31)2077	79)79	5)25	29	3)135
67)67	1	5)5		3)45
1	Ans. 11, 23, 79.	1		3)15
Ans. 2, 3, 31, 67.		Ans. 3, 5.		5)5
				1

```
(9.) 2)960        2)1568        2)5824
     2)480        2)784         2)2912
     2)240        2)392         2)1456
     2)120        2)196         2)728
     2)60         2)98          2)364
     2)30         7)49          2)182
     3)15         7)7           7)91
     5)5          1             13)13
     1        Ans. 2.          1

(10.) 2)2340   2)11934   3)12987   3)14859
      2)1170   3)5967    3)4329    3)4953
      3)585    3)1989    3)1443    13)1651
      3)195    3)663     13)481    127)127
      5)65     13)221    37)37     1
      13)13    17)17     1
      1        1      Ans. 3, 3, 13.

      (11.) 3)105
            5)35
            7)7
            1    Ans. 3, 5, 7.
```

DIVISORS OR MEASURES.

(Art. 194, p. 145.)

(2.) $72 = 2 \times 2 \times 2 \times 3 \times 3$.

Divisors. $\begin{cases} 1, 2, 4, 8 & = 2 \times 2 \times 4. \\ 3, 6, 12, 24 & = (2 \times 2 \times 2) \times 3. \\ 9, 18, 36, 72 & = (2 \times 2 \times 2) \times (3 \times 3). \end{cases}$

(3.) $105 = 3 \times 5 \times 7$.

Exponent of 3 is $1 + 1 = 2$.

Exponent of 5 is $1 + 1 = 2$.

Exponent of 7 is $1 + 1 = 2$.

$2 \times 2 \times 2 = 8$, the number of divisors.

4. $1764 = 2 \times 2 \times 3 \times 3 \times 7 \times 7.$

2 (the exponent of factor 2) $+1 = 3$; $2 + 1 = 3$; $2+1 = 3$; $3 \times 3 \times 3 = 27$, Ans.

5. $3528 = 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7 =$

3 (the exponent of the factor 2) $+1 = 4$; $2 + 1 = 3$; $2 + 1 = 3$; $4 \times 3 \times 3 = 36$, Ans.

6. $5880 = 2 \times 2 \times 2 \times 3 \times 5 \times 7 \times 7.$

$3 + 1 = 4$; $1 + 1 = 2$; $1 + 1 = 2$; $2 + 1 = 3$; $4 \times 2 \times 2 \times 3 = 48$, Ans.

COMMON DIVISORS.

(Art. **198,** p. **146.**)

2. $51 = 3 \times 17.$

$153 = 3 \times 3 \times 17.$

$255 = 3 \times 5 \times 17.$

The prime factors which are common are 1, 3, 17.   Their products are found as in Art. **194.**

Divisors 1, 3, 17, 51 = Ans. 1, 3, 17, 51.

3. $180 = 2 \times 2 \times 3 \times 3 \times 5.$

$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5.$

2, 2, 3, 3, 5, are common.   Hence,

1,   2,   4,

3,   6, 12 $= (2 \times 2) \times 3.$

5, 10, 20 $= (2 \times 2) \times 5.$

9, 18, 36 $= (2 \times 2) \times (3 \times 3).$

15, 30, 60 $= (2 \times 2) \times (3 \times 5).$

45, 90, 180 $= (2 \times 2) \times (3 \times 3 \times 5).$

Ans. 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 45,
[60, 90, 180.

4. $2025 = 3 \times 3 \times 3 \times 3 \times 5 \times 5.$

$6075 = 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5.$

$8100 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5.$

The factors $3 \times 3 \times 3 \times 3 \times 5 \times 5$ are common.

The exponent of 3 is $4 + 1 = 5$; the exponent of 5 is $2 + 1 = 3$; and $5 \times 3 = 15$, Ans.

5*

5. $4500 = 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5.$
   $9000 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5.$

The *common* prime factors are $2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5.$
Their exponents are $2 + 1 = 3$; $2 + 1 = 3$; $3 + 1 = 4$; and
$3 \times 3 \times 4 = 36$, Ans.

(Art. 200, p. 148.)

THE GREATEST COMMON DIVISOR OR MEASURE.

2.	56	6.	35	10.	15
3.	32	7.	1	11.	78
4.	57	8.	1	12.	4 acre lots.
5.	1	9.	12	13.	6 bushels.

LEAST COMMON MULTIPLE.

(Art. 203, p. 151.)

(2.) $\underline{3 \quad 13 \quad 37 \mid 91}$
      $\quad 3 \quad 1 \quad 37$

$3 \times 1 \times 37 \times 91 = 10101.$

(3.) $3) \underline{9 \quad 14 \quad 30 \quad 35 \quad 47}$
      $5) \underline{3 \quad 14 \quad 10 \quad 35 \quad 47}$
      $2) \underline{3 \quad 14 \quad 2 \quad 7 \quad 47}$
      $7) \underline{3 \quad 7 \quad 1 \quad 7 \quad 47}$
      $\quad \; 3 \quad 1 \quad 1 \quad 1 \quad 47$

$47 \times 3 \times 7 \times 2 \times 5 \times 3 = 29610$, Ans.

(4.) $\underline{6 \quad 8 \quad 10 \quad 18 \quad 20 \mid 24}$
      $\qquad\qquad 3 \quad 5$

$3 \times 5 \times 24 = 360$, Ans.

(5.) $\underline{14 \quad 19 \quad 38 \mid 57}$
      $\quad 7 \qquad 2$

$7 \times 2 \times 57 = 798$, Ans.

(6.) $\underline{20 \quad 36 \quad 48 \mid 50}$
      $\quad \underline{2 \quad 18 \mid 24}$
      $\quad \; 1 \quad 3$

$3 \times 24 \times 50 = 3600$, Ans.

(7.) $\underline{15 \quad 25 \quad 35 \quad 45 \mid 100}$
      $\qquad\qquad 7 \quad 9$

$7 \times 9 \times 100 = 6300$, Ans.

(8.) $\underline{100 \quad 200 \quad 300 \quad 400 \mid 575}$
      $\qquad\qquad \underline{12 \mid 16}$
      $\qquad\qquad \; 3$

$3 \times 16 \times 575 = 27600$, Ans.

(9.) 1̶ 2̶ 3̶ 4̶ 5 6 8 | 9
           5̶ 2̶ 8

$5 \times 8 \times 9 = 360; \ 2520 \div 360 = 7$, **Ans.**

(10.) 1̶8̶ 2̶4̶ 36 20 48 | 126
             2̶ 10 | 8
                5

$5 \times 8 \times 126 = 5040$, **Ans.**

(11.) 4̶ 6̶ 10 | 12
           5

$5 \times 12 = 60$qt., **Ans.**

(12.) 50 40 | 75
         2̶ 8

$8 \times 75 = 600$, **Ans.**

MISCELLANEOUS EXAMPLES.

(Page 152.)

(1.) 7)6174
     7)882
     7)126
     2)18
     3)9
     3)3
        1   Ans. 3 times.

(2.) 5)5775
     5)1155
     7)231
     3)33
     11)11
         1   11, **Ans.**

(3.) 3)19929
     7)6643
     13)949
     73)73
        1

$73 \times 13 \times 7 = 6643$, **Ans.**

(4.) 2)2338
     7)1169
     167)167
          1

$2338 \div 2 = 1169$ }
$2338 \div 7 = 334$ } **Ans.**

(5.) 5)17885
3)3477
19)1159
61)61
1

Ans. 3, 5, 19, 61.

(6.) 2)3000 $5 \times 5 \times 5 \times 3 \times 2 \times 2 = 1500$
3)1500 $5 \times 5 \times 5 \times 2 \times 2 \times 2 = 1000$ } Bins.
5)500 $5 \times 5 \times 5 \times 3 \times 2 \quad = 750$
2)100
2)50   1, 2, 3 = Bags.
5)25   Ans.
5)5
1

(7.) 136)152(1 $152 \div 8 = 19$ classes of ladies.
136        $136 \div 8 = 17$ classes of gentlemen.
16)136(8   Ans.
128
8)16(2
16

8. The velocity of the hands of a clock is as the times in which they perform one revolution. The second-hand performs a revolution in 60 seconds, the minute-hand in 3600 seconds, and the hour-hand in 43200 seconds.

60)43200 3600 60
60) 720   60   1
12   1   1

$60 \times 60 \times 12 = 43200$ seconds = 12 hours, Ans.

9. Greatest common divisor of 348, 372, 444, 492 = 12; $348 \div 12 = 29$ rails; $372 \div 12 = 31$ rails; $444 \div 12 = 37$ rails; $492 \div 12 = 41$ rails; $29 + 31 + 37 + 41 = 138$ rails; $138 \times 5 = 690$ rails; 12ft. + 9in. = 12ft. 9in., length of each rail, Ans.

10. 3A. 2R. 1p. = 561p.; 5A. 3R. 15p. = 935p.; 8A. 0R.
    29p. = 1309p.; 12A. 3R. 17p. = 2057p.; 15A. 0R.
    31p. = 2431p.; 1309 ÷ 187 = 7; 2057 ÷ 187 = 11;
    187p. is therefore the common divisor, and each field
    will contain 187p. = 1A. 0R. 27p., Ans.

$$561)935(1$$
$$\underline{561}$$
$$374)561(1$$
$$\underline{374}$$
$$187)374(2$$
$$\underline{374}$$

(11.) $2772 = 2 \times 2 \times 3 \times 3 \times 7 \times 11$
     $12 = 2 \times 2 \times 3$
     $12 \times 3 = 36$
     $12 \times 7 = 84$ } Ans.
     $12 \times 11 = 132$

(12.) $3 + 1 = 4; 5 + 1 = 6; 7 + 1 = 8$

$$2 \underline{)4 \quad 6 \quad 8}$$
$$2 \underline{)2 \quad {}^{\cdot}3 \quad 4}$$
$$1 \quad 3 \quad 2$$

$2 \times 2 \times 3 \times 2 = 24; 24 - 1 = 23$ days, Ans.

COMMON FRACTIONS.

REDUCTION.

(Art. **220,** p. **156.**)

2.	$\frac{4}{21}$	5.	$\frac{7}{54}$	8.	$\frac{1}{7}$	11.	$\frac{41}{61}$
3.	$\frac{3}{4}$	6.	$\frac{3}{35}$	9.	$\frac{418}{575}$	12.	$\frac{408}{509}$
4.	$\frac{3}{7}$	7.	$\frac{81}{422}$	10.	$\frac{811}{1116}$	13.	$\frac{41}{152}$

(Art. **221,** p. **157.**)

2.	$11\frac{2}{15}$	5.	$3\frac{134}{181}$	8.	125
3.	$14\frac{7}{116}$	6.	$111\frac{1}{8}$	9.	1
4.	$7\frac{12}{17}$	7.	$91\frac{23}{45}$		

(Art. **222**, p. **157.**)

3.	$\frac{80}{4}$	7.	$\frac{102}{1}$	10.	$\frac{7107}{61}$	13.	$\frac{7}{1}$
4.	$2\frac{17}{16}$	8.	$\frac{85}{17}$	11.	$\frac{68691}{97}$	14.	$24\frac{7}{13}$
5.	$10\frac{18}{59}$	9.	$25\frac{24}{37}$	12.	$20\frac{202}{199}$	15.	$46\frac{6}{4}$
6.	$\frac{11}{1}$						

(Art. **223**, p. **158.**)

3. $\frac{2}{3} \times \frac{5}{6} \times \frac{7}{8} \times \frac{11}{12} = \frac{770}{1728} = \frac{385}{864}$, Ans.

4. $\frac{\cancel{6}}{7} \times \frac{11}{\cancel{15}} \times \frac{\cancel{3}}{5} \times \frac{1}{\cancel{18}} = \frac{11}{525}$, Ans.

5. $\frac{\cancel{4}}{7} \times \frac{5}{\cancel{8}} \times \frac{3}{8} \times \frac{11}{13} = \frac{165}{1456}$, Ans.

6. $\frac{6}{11} \times \frac{6}{\cancel{7}} \times \frac{1}{4} \times \frac{\cancel{21}}{1} = \frac{108}{44} = 2\frac{5}{11}$, Ans.

7. $\frac{7}{11} \times 15\frac{7}{8} \times 5\frac{7}{10} \times 100 = \frac{7}{11} \times \frac{127}{8} \times \frac{57}{\cancel{10}} \times \frac{\cancel{100}}{1} =$

$\left[\frac{506730}{88} = 5758\frac{13}{44}\right.$, Ans.

8. $\frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{7} \times \frac{11}{\cancel{24}} = \frac{11}{56}$, Ans.

9. $\frac{7}{\cancel{11}} \times \frac{\cancel{11}}{\cancel{25}} \times \frac{\cancel{25}}{31} \times \frac{7.75}{1} = \frac{54.25}{31} = \$1.75$, Ans.

10. $\frac{4}{9} \times \frac{9}{17} \times \frac{17}{18} \times 3\frac{3}{5} = \frac{4}{\cancel{9}} \times \frac{\cancel{9}}{\cancel{17}} \times \frac{\cancel{17}}{\cancel{18}} \times \frac{\cancel{18}}{5} = \frac{4}{5}$ gal., Ans.

11. $\frac{1}{\cancel{4}} \times \frac{\cancel{2}}{3} \times \frac{\cancel{3}}{4} = \frac{1}{8}$, Ans.

12. $\dfrac{4}{7} \times \dfrac{9}{20} \times \dfrac{15}{16} \times \dfrac{14}{17} \times \dfrac{34}{1} = \dfrac{108}{16} = \$6.75$, Ans.

COMMON DENOMINATORS.

(Art. 226, p. 160.)

2. $\frac{54}{72}, \frac{48}{72}, \frac{63}{72}, \frac{30}{72}.$

3. $\frac{720}{1320}, \frac{704}{1320}, \frac{1254}{1320}, \frac{275}{1320}.$

4. $\frac{12}{84}, \frac{18}{84}, \frac{32}{84}, \frac{15}{64}.$

5. $\frac{22}{42}, \frac{24}{42}, \frac{33}{42}, \frac{21}{42}.$

6. $\frac{210}{308}, \frac{84}{308}, \frac{176}{308}, \frac{231}{308}.$

7. $\frac{30}{60}, \frac{40}{60}, \frac{45}{60}, \frac{48}{60}.$

8. $\frac{594}{693}, \frac{228}{693}, \frac{315}{693}, \frac{441}{693}.$

9. $\frac{80}{120}, \frac{105}{120}, \frac{20}{120}, \frac{156}{120}.$

10. $\frac{70}{168}, \frac{63}{168}, \frac{96}{168}, \frac{798}{168}.$

11. $\frac{24}{42}, \frac{15}{42}, \frac{22}{42}, \frac{21}{42}.$

12. $\frac{224}{252}, \frac{147}{252}, \frac{288}{252}, \frac{24}{252}.$

13. $\frac{570}{600}, \frac{140}{600}, \frac{165}{600}, \frac{12}{600}.$

14. $\frac{318}{240}, \frac{148}{240}, \frac{125}{240}, \frac{84}{240}.$

15. $\frac{14}{63}, \frac{441}{63}, \frac{504}{63}, \frac{324}{63}.$

16. $\frac{80}{120}, \frac{96}{120}, \frac{105}{120}.$

17. $\frac{360}{630}, \frac{560}{630}, \frac{189}{630}.$

18. $\frac{546}{1001}, \frac{572}{1001}, \frac{616}{1001}.$

19. $\frac{864}{624}, \frac{192}{624}, \frac{4836}{624}.$

20. $\frac{1485}{2295}, \frac{1020}{2295}, \frac{612}{2295}.$

21. $\frac{2940}{2295}, \frac{540}{2295}, \frac{26316}{2295}.$

22. $\frac{21}{42}, \frac{28}{42}, \frac{24}{42}, \frac{336}{42}.$

23. $\frac{528}{1188}, \frac{756}{1188}, \frac{6039}{1188}.$

24. $\frac{24}{48}, \frac{36}{48}, \frac{8}{48}, \frac{816}{48}.$

25. $\frac{110}{120}, \frac{576}{120}, \frac{2580}{120}.$

26. $\frac{12012}{14014}, \frac{5096}{14014}, \frac{5390}{14014},$ $\left[\frac{8008}{14014}, \frac{7007}{14014}.\right.$

27. $\frac{285068816}{178431552}, \frac{37088064}{178431552},$ $\left[\frac{722813}{178431552}.\right.$

ADDITION OF COMMON FRACTIONS.

(Art. 227, p. 162.)

3.	$3\frac{10}{17}.$	11.		1.	18.	$9\frac{115}{252}.$
4.	$2\frac{10}{23}.$	12.		$3\frac{19}{60}.$	19.	$18\frac{311}{336}.$
5.	$2\frac{2}{87}.$	13.		$1\frac{83}{140}.$	20.	$45\frac{1}{126}.$
6.	$1\frac{2}{3}.$	14.		$6\frac{342}{504}.$	21.	$40\frac{4}{15}.$
7.	$2\frac{136}{231}.$	15.		$9\frac{21}{24}.$	22.	$1068\frac{37}{45}.$
8.	$2\frac{43}{72}.$	16.		$6\frac{1}{4}.$	23.	$12\frac{591}{835}.$
9.	$1\frac{221}{264}.$	17.		$17\frac{7}{20}.$	24.	$40\frac{4}{15}.$
10.	$2\frac{133}{360}.$					

SUBTRACTION OF COMMON FRACTIONS

(Art. 230, p. 165.)

3.	$\frac{7}{17}$	11.	$\frac{7}{40}$	19.	$\frac{8}{25}$.
4.	$\frac{9}{19}$	12.	$\frac{69}{128}$	20.	$\frac{33}{100}$.
5.	$\frac{15}{37}$	13.	$\frac{13}{250}$	21.	$\frac{5}{54}$.
6.	$\frac{2}{39}$	14.	$\frac{19}{34}$	22.	$\frac{3}{55}$.
7.	$\frac{5}{29}$	15.	$\frac{17}{60}$	23.	$1\frac{9}{28}$.
8.	$\frac{15}{37}$	16.	$\frac{124}{325}$	24.	$1\frac{1}{12}$.
9.	$\frac{1}{4}$	17.	$\frac{627}{732}$	25.	$\frac{1}{12}$.
10.	$\frac{5}{36}$	18.	$\frac{92}{231}$		

(Art. 231, p. 166.)

2.	$26\frac{5}{8}$.	6.	$15\frac{139}{237}$.	10.	$77\frac{9}{23}$.
3.	$11\frac{1}{4}$.	7.	$16\frac{1}{4}$.	11.	$680\frac{9}{44}$.
4.	$670\frac{9}{50}$.	8.	$5\frac{4}{7}$.	12.	$30\frac{1}{4}$.
5.	$368\frac{42}{39}$.	9.	$18\frac{4}{7}$.	13.	$31\frac{8}{19}$.

(Art. 232, p. 167.)

$$
\begin{array}{ccc}
(2.) & (3.) & (4.) \\
18\frac{3}{8} = 18\frac{27}{72} & 73\frac{4}{5} = 73\frac{48}{60} & 67\frac{1}{3} = 67\frac{3}{9} \\
9\frac{8}{9} = 9\frac{64}{72} & 16\frac{11}{12} = 16\frac{55}{60} & 16\frac{2}{3} = 16\frac{6}{9} \\
\hline
8\frac{35}{72}, \text{Ans.} & 56\frac{53}{60}, \text{Ans.} & 50\frac{2}{3}, \text{Ans.}
\end{array}
$$

$$
\begin{array}{ccc}
(5.) & (6.) & (7.) \\
29\frac{18}{19} = 29\frac{162}{171} & 144\frac{3}{4} = 144\frac{39}{52} & 171\frac{15}{23} = 171\frac{150}{230} \\
15\frac{4}{9} = 15\frac{76}{171} & 99\frac{11}{13} = 99\frac{44}{52} & 91\frac{9}{10} = 91\frac{168}{230} \\
\hline
14\frac{86}{171}, \text{Ans.} & 44\frac{47}{52}, \text{Ans.} & 79\frac{13}{55}, \text{Ans.}
\end{array}
$$

$$
\begin{array}{ccc}
(8.) & (9.) & (10.) \\
101\frac{7}{8} = 101\frac{35}{40} & 165\frac{1}{2} = 165\frac{8}{16} & 77\frac{11}{12} = 77\frac{220}{240} \\
93\frac{5}{8} = 93\frac{32}{40} & 98\frac{7}{8} = 98\frac{14}{16} & 19\frac{19}{20} = 19\frac{228}{240} \\
\hline
8\frac{3}{40}, \text{Ans.} & 66\frac{5}{8}, \text{Ans.} & 57\frac{232}{240}, \text{Ans.}
\end{array}
$$

(11.)	(12.)	(13.)
$84\frac{23}{24} = 84\frac{276}{288}$	$671\frac{11}{12} = 671\frac{9213}{9843}$	$17\frac{189}{457} = 17\frac{87453}{324927}$
$15\frac{11}{12} = 15\frac{264}{288}$	$183\frac{69}{83} = 183\frac{48748}{58843}$	$8\frac{744}{711} = 8\frac{106938}{324927}$
$69\frac{12}{288}$, Ans.	$487\frac{8397}{9843}$, Ans.	Ans. $8\frac{30144}{324927}$

14. $63 - 7\frac{9}{11} = 55\frac{2}{11}$ gal., Ans.

15. $30 - 5\frac{7}{12} = 24\frac{5}{12}$ days, Ans.

16 $17\frac{5}{8} + 28\frac{7}{12} = 46\frac{5}{24}$; $144 - 46\frac{5}{24} = 97\frac{19}{24}$ lb, Ans.

17. $34 - 9\frac{7}{8} = 24\frac{1}{8}$ yd., Ans.

18. $40 \times \frac{2}{17} = 4\frac{12}{17}$; $40 \times \frac{3}{11} = 7\frac{3}{11}$; $4\frac{12}{17} + 7\frac{3}{11} = 11\frac{183}{187}$; $40 - 11\frac{183}{187} = 28\frac{4}{187}$ miles, Ans.

19. $\frac{1}{4}$ of a square yard $= \frac{1}{4}$ of 9 feet $= 2\frac{1}{4}$ square feet. 1 yard $= 3$ feet; $\frac{1}{6}$ of 3 feet $= 6$ inches. 6 inches squared $= 36$ square inches $= \frac{1}{4}$ of a square foot. $2\frac{1}{4}$ feet $- \frac{1}{4}$ foot $= 2$ square feet, Ans.

MISCELLANEOUS EXAMPLES IN ADDITION AND SUBTRACTION OF FRACTIONS.

(Page 168.)

1. $\frac{3}{4}, \frac{1}{2}, \frac{7}{8} = \frac{6}{8}, \frac{4}{8}, \frac{7}{8}$; $6 + 4 + 7 = 17$; $17 \div 8 = 2\frac{1}{8}$, Ans.

2. $\frac{5}{8} + \frac{1}{2} + \frac{1}{4} + \frac{5}{6} = 1\frac{5}{24} + 1\frac{2}{24} + 2\frac{9}{24} + 2\frac{20}{24} = \frac{15+12+8+20}{24} = \frac{55}{24} = \$2\frac{7}{24}$, Ans.

3. $20\frac{7}{10}, 19\frac{1}{2}, 22\frac{1}{16} = 20\frac{56}{80}, 19\frac{40}{80}, 22\frac{5}{80}$; $20\frac{56}{80} + 19\frac{40}{80} + 22\frac{5}{80} = 62\frac{21}{80}$ miles, Ans.

4. $6\frac{1}{2}, 19\frac{4}{5}, 3\frac{3}{4} = 6\frac{10}{20}, 19\frac{16}{20}, 3\frac{15}{20}$; $6\frac{10}{20} + 19\frac{16}{20} + 3\frac{15}{20} = 30\frac{1}{20}$ tons, Ans.

5. $\frac{1}{5}, \frac{1}{3} = \frac{3}{15}, \frac{5}{15}$; $\frac{3}{15} + \frac{5}{15} = \frac{8}{15}$; $1 - \frac{8}{15} = \frac{15}{15} - \frac{8}{15} = \frac{7}{15}$, Ans.

6. $1 - \frac{2}{3} = \frac{3}{3} - \frac{2}{3} = \frac{1}{3}$; $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$; $\frac{2}{3} + \frac{1}{12} = \frac{8}{12} + \frac{1}{12} = \frac{9}{12} = \frac{3}{4}$; $1 - \frac{3}{4} = \frac{1}{4}$; $\frac{4}{4} - \frac{3}{4} = \frac{1}{4}$, Ans.

7. $11\frac{5}{8}, 9\frac{1}{2} = 11\frac{5}{8}, 9\frac{4}{8}$; $11\frac{5}{8} + 9\frac{4}{8} = 21\frac{1}{8}$; $31\frac{1}{4}, 21\frac{1}{8} = 31\frac{2}{8}$, $21\frac{1}{8}$; $31\frac{2}{8} - 21\frac{1}{8} = 10\frac{1}{8}$ yards, Ans.

6

8. $4\frac{3}{7}$, 11, $26\frac{1}{2} = 4\frac{6}{14}$, 11, $26\frac{7}{14}$; $4\frac{6}{14} + 11 + 26\frac{7}{14} = 41\frac{13}{14}$; $84\frac{3}{8} - 41\frac{13}{14} = 42\frac{25}{56}$, $+ 3\frac{3}{4} = 46\frac{11}{56}$; $46\frac{11}{56} - 13\frac{8}{11} =$ $46\frac{121}{616} - 13\frac{448}{616} = 32\frac{289}{616}$; $32\frac{289}{616} + 3\frac{1}{2} = 32\frac{289}{616} +$ $3\frac{308}{616} = 35\frac{597}{616}$ gallons, Ans.

9. $19\frac{3}{4}$, $36\frac{1}{3}$, $33\frac{3}{8} = 19\frac{18}{24}$, $36\frac{8}{24}$, $33\frac{9}{24} = 19\frac{18}{24} + 36\frac{8}{24} +$ $33\frac{9}{24} = 89\frac{11}{24}$; $89\frac{11}{24} - 71\frac{3}{8} = 89\frac{11}{24} - 71\frac{9}{24} = 18\frac{1}{3}$ yards, Ans.

MULTIPLICATION OF COMMON FRACTIONS.

(ART. 235, p. 171.)

5. $12 \times \frac{5}{7} = \frac{12}{1} \times \frac{5}{7} = \frac{60}{7} = 8\frac{4}{7}$, Ans.

6. $\frac{2}{7} \times 12 = \frac{2}{7} \times \frac{12}{1} = \frac{24}{7} = 3\frac{3}{7}$, Ans.

7. $\frac{14}{17} \times \frac{17}{42} = \frac{\overset{1}{\cancel{14}}}{\cancel{17}} \times \frac{\cancel{17}}{\underset{3}{\cancel{42}}} = \frac{1}{3}$, Ans.

8. $\frac{3}{7} \times \frac{19}{20} = \frac{57}{140}$, Ans.

9. $\frac{4}{5} \times \frac{6}{11} = \frac{24}{55}$, Ans.

10. $\frac{7}{12} \times \frac{11}{16} = \frac{77}{192}$, Ans.

11. $\frac{8}{11} \times \frac{4}{7} = \frac{32}{77}$, Ans.

12. $\frac{3}{4} \times \frac{15}{16} = \frac{45}{64}$, Ans.

13. $\frac{16}{17} \times \frac{11}{12} = \frac{\overset{4}{\cancel{16}}}{17} \times \frac{11}{\underset{3}{\cancel{12}}} = \frac{44}{51}$, Ans.

14. $\frac{9}{10} \times \frac{14}{1} = \frac{9}{\cancel{10}} \times \frac{\overset{7}{\cancel{14}}}{1} = \frac{63}{5} = 12\frac{3}{5}$, Ans.

15. $13 \times \frac{4}{7} = \frac{13}{1} \times \frac{4}{7} = \frac{52}{7} = 7\frac{3}{7}$, Ans.

16. $16 \times \frac{3}{19} = \frac{16}{1} \times \frac{3}{19} = \frac{48}{19} = 2\frac{10}{19}$, Ans.

17. $11 \times \frac{4}{7} = \frac{11}{1} \times \frac{4}{7} = \frac{44}{7} = 6\frac{2}{7}$, Ans.

18. $\frac{9}{10} \times \frac{14}{1} = \frac{9}{\cancel{10}} \times \frac{\overset{7}{\cancel{14}}}{1} = \frac{63}{5} = 12\frac{3}{5}$, Ans.

19. $\frac{8}{9} \times \frac{19}{1} = \frac{152}{9} = 16\frac{8}{9}$, Ans.

20. $\frac{9}{11} \times \frac{22}{27} = \frac{\overset{1}{\cancel{9}}}{\underset{1}{\cancel{11}}} \times \frac{\overset{2}{\cancel{22}}}{\underset{3}{\cancel{27}}} = \frac{2}{3}$, Ans.

21. $\frac{4}{9} \times \frac{18}{19} = \frac{4}{\underset{1}{\cancel{9}}} \times \frac{\overset{2}{\cancel{18}}}{19} = \frac{8}{19}$, Ans.

22. $\frac{3}{19} \times \frac{38}{39} = \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{19}}} \times \frac{\overset{2}{\cancel{38}}}{\underset{13}{\cancel{39}}} = \frac{2}{13}$, Ans.

23. $\frac{7}{9} \times \frac{9}{11} = \frac{7}{\cancel{9}} \times \frac{\cancel{9}}{11} = \frac{7}{11}$, Ans.

24. $\frac{1}{99} \times \frac{99}{100} = \frac{1}{\cancel{99}} \times \frac{\cancel{99}}{100} = \frac{1}{100}$, Ans.

25. $\frac{3}{7} \times \frac{7}{11} \times \frac{11}{24} + 100 = \frac{\cancel{3}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{11}} \times \frac{\cancel{11}}{\underset{8}{\cancel{24}}} \times \frac{\overset{1}{100}}{1} = \frac{100}{8} = 12\frac{1}{2}$, [Ans.

26. $\frac{1}{2} \times \frac{2}{3} \times \frac{7}{8} \times \frac{11}{1} = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3} \times \frac{7}{8} \times \frac{11}{1} = \frac{77}{24} = 3\frac{5}{24}$, Ans.

27. $\frac{7}{12} \times \frac{17}{1} = \frac{119}{12} = \$9\frac{11}{12}$, Ans.

28. $\frac{9}{20} \times \frac{37}{1} = \frac{333}{20} = \$16\frac{13}{20}$, Ans.

29. $\frac{5}{\cancel{8}} \times \frac{\cancel{8}}{1} \times \frac{7}{1} = \$35$, Ans.

30. $161\frac{11}{15} \times 19\frac{11}{28} = \frac{2426}{15} \times \frac{543}{28} = \frac{1317318}{420} = 3136\frac{33}{70}$, Ans.

31. $\frac{3}{7} \times 8\frac{3}{4} = \frac{3}{7} \times \frac{35}{4} = \frac{105}{28} = 3\frac{3}{4}$, Ans.

32. $\frac{9}{10} \times 17\frac{3}{11} = \frac{9}{10} \times \frac{190}{11} = \frac{1710}{110} = 15\frac{6}{11}$, Ans.

33. $\frac{8}{9} \times 71\frac{4}{5} = \frac{8}{9} \times \frac{359}{5} = \frac{2872}{45} = 63\frac{37}{45}$, Ans.

34. $\frac{3}{4} \times 9\frac{1}{4} \times \frac{2}{3} \times 17 = \frac{\cancel{3}}{4} \times \frac{37}{\underset{2}{\cancel{4}}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{17}{1} = \frac{629}{8} = 78\frac{5}{8}$, [Ans.

35. $\frac{9}{10} \times 7 \times \frac{11}{15} \times 87\frac{3}{11} = \frac{\overset{3}{\cancel{9}}}{\cancel{10}} \times \frac{7}{1} \times \frac{\cancel{11}}{\cancel{15}} \times \frac{\overset{96}{\cancel{960}}}{\cancel{11}} = \frac{2016}{5} = $ [403$\frac{1}{5}$, Ans.

36. $8 \times \frac{7}{9} = \frac{8}{1} \times \frac{7}{9} = \frac{56}{9} = 6\frac{2}{9}$, Ans.

37. $12 \times \frac{5}{7} = \frac{12}{1} \times \frac{5}{7} = \frac{60}{7} = 8\frac{4}{7}$, Ans.

38. $15 \times \dfrac{6}{11} = \dfrac{15}{1} \times \dfrac{6}{11} = \dfrac{90}{11} = 8\frac{2}{11}$, Ans.

39. $\dfrac{7}{\overset{8}{\underset{2}{}}} \times \dfrac{4}{11} = \dfrac{7}{22}$, Ans.

40. $3\frac{7}{9} \times 10\frac{4}{7} = \dfrac{34}{9} \times \dfrac{74}{7} = \dfrac{2516}{63} = 39\frac{59}{63}$, Ans.

41. $\dfrac{2}{3} \times 7\frac{1}{4} \times \dfrac{7}{8} \times 11\frac{3}{4} = \dfrac{2}{3} \times \dfrac{29}{4} \times \dfrac{7}{8} \times \dfrac{47}{4} = \dfrac{9541}{192} = 49\frac{133}{192}$,

[Ans.

42. $\dfrac{2}{7} \times 9 \times \dfrac{3}{5} \times 17 = \dfrac{2}{7} \times \dfrac{9}{1} \times \dfrac{3}{5} \times \dfrac{17}{1} = \dfrac{918}{35} = 26\frac{8}{35}$, Ans.

43. $\dfrac{4}{7} \times 8\frac{3}{10} \times \dfrac{4}{7} \times 9\frac{1}{4} = \dfrac{4}{7} \times \dfrac{83}{10} \times \dfrac{4}{7} \times \dfrac{37}{4} = \dfrac{12284}{490} = 25\frac{17}{245}$,

[Ans.

(ART. 236, p. 172.)

(3.)	(4.)	(5.)

	(3.)		(4.)		(5.)
	$8\frac{2}{5}$		$3\frac{1}{8}$		$8\frac{4}{7}$
	7		17		13
$\frac{3}{5} \times 7 =$	$4\frac{1}{5}$	$\frac{1}{8} \times 17 =$	$2\frac{1}{8}$	$\frac{4}{7} \times 13 =$	$5\frac{4}{7}$
$7 \times 8 =$	56	$3 \times 17 =$	41	$8 \times 13 =$	104
	$60\frac{1}{5}$		$43\frac{1}{8}$		$109\frac{4}{7}$

	(6.)		(7.)		(8.)
	$13\frac{8}{11}$		$11\frac{6}{7}$		$7\frac{6}{11}$
	37		8		5
$\frac{8}{11} \times 37 =$	$26\frac{10}{11}$	$\frac{6}{7} \times 8 =$	$6\frac{6}{7}$	$\frac{6}{11} \times 5 =$	$2\frac{8}{11}$
$37 \times 13 =$	481	$8 \times 11 =$	88	$5 \times 7 =$	35
	$507\frac{10}{11}$		$94\frac{6}{7}$		$\$ 0.37\frac{8}{11}$

6*

(9.)	(10.)	(11.)
$23\frac{7}{12}$	$8\frac{3}{8}$	$6\frac{3}{8}$
6	5	9
$\frac{7}{12} \times 6 = \overline{3\frac{1}{2}}$	$\frac{3}{8} \times 5 = \overline{1\frac{7}{8}}$	$\frac{3}{8} \times 9 = \overline{3\frac{3}{8}}$
$6 \times 23 = 138$	$8 \times 5 = 40$	$9 \times 6 = 54$
$\$141\frac{1}{2}$	$\$41\frac{7}{8}$	$\$57\frac{3}{8}$

(12.)	(13.)	(14.)
$\$6.37\frac{1}{2}$	$9\frac{3}{8}$	$4\frac{3}{8}$
12	11	1.75
$\frac{1}{2} \times 12 = \overline{6}$	$\frac{3}{8} \times 11 = \overline{4\frac{1}{8}}$	$\frac{3}{8} \times 1.75 = \overline{65\frac{5}{8}}$
$12 \times 6.37 = 76.44$	$11 \times 9 = 99$	$1.75 \times 4 = 700$
$\$76.50$	$\$103\frac{1}{8}$	$\$7.65\frac{5}{8}$

(15.)	(16.)	(17.)
$11\frac{7}{8}$	$\$10\frac{5}{8}$	$\$3\frac{1}{8}$
7	9	5
$\frac{7}{8} \times 7 = \overline{6\frac{1}{8}}$	$\frac{5}{8} \times 9 = \overline{5\frac{5}{8}}$	$\frac{1}{8} \times 5 = \overline{0\frac{5}{8}}$
$7 \times 11 = 77$	$9 \times 10 = 90$	$5 \times 3 = 15$
$\$83\frac{1}{8}$	$\$95\frac{5}{8}$	$\$15\frac{5}{8}$

(18.)	(19.)
$\$7.62\frac{1}{2}$	$\$8.37\frac{1}{2}$
15	40
$\frac{1}{2} \times 15 = \overline{7\frac{1}{2}}$	$\frac{1}{2} \times 40 = \overline{20}$
$15 \times 7.62 = 114.30$	$40 \times 8.37 = 334.80$
$\$114.37\frac{1}{2}$	$\$335.00$

DIVISION OF COMMON FRACTIONS.

(ART. 239, p. 175.)

6. $\dfrac{7}{11} \div 18 = \dfrac{7}{11} \times \dfrac{1}{18} = \dfrac{7}{198}$, Ans.

7. $\dfrac{4}{9} \div \dfrac{7}{8} = \dfrac{4}{9} \times \dfrac{8}{7} = \dfrac{32}{63}$, Ans.

8. $18 \div \frac{7}{11} = \frac{18}{1} \times \frac{11}{7} = \frac{198}{7} = 28\frac{2}{7}$, Ans.

9. $\frac{5}{27} \div \frac{5}{3} = \frac{5}{27} \times \frac{3}{5} = \frac{15}{135} = \frac{1}{9}$, Ans.

10. $\frac{16}{21} \div \frac{2}{3} = \frac{8}{7} = 1\frac{1}{7}$, Ans.

11. $\frac{15}{22} \div 28 = \frac{15}{22} \times \frac{1}{28} = \frac{15}{616}$, Ans.

12. $\frac{1}{17} \div 27 = \frac{1}{17} \times \frac{1}{27} = \frac{1}{459}$, Ans.

13. $\frac{2}{15} \div 128 = \frac{\overset{1}{\cancel{2}}}{15} \times \frac{1}{\underset{64}{\cancel{128}}} = \frac{1}{960}$, Ans.

14. $\frac{11}{17} \div 98 = \frac{11}{17} \times \frac{1}{98} = \frac{11}{1666}$, Ans.

15. $\frac{14}{23} \div 19 = \frac{14}{23} \times \frac{1}{19} = \frac{14}{437}$, Ans.

16. $\frac{5}{6} \div 167 = \frac{5}{6} \times \frac{1}{167} = \frac{5}{1002}$, Ans.

17. $\frac{16}{29} \div 49 = \frac{16}{29} \times \frac{1}{49} = \frac{16}{1421}$, Ans.

18. $\frac{1}{15} \div 15 = \frac{1}{15} \times \frac{1}{15} = \frac{1}{225}$, Ans.

19. $27 \div \frac{1}{17} = \frac{27}{1} \times \frac{17}{1} = 459$, Ans.

20. $128 \div \frac{2}{15} = \frac{\overset{64}{\cancel{128}}}{1} \times \frac{15}{\underset{1}{\cancel{2}}} = 960$, Ans.

21. $98 \div \frac{11}{17} = \frac{98}{1} \times \frac{17}{11} = \frac{1666}{11} = 151\frac{5}{11}$, Ans.

22. $19 \div \frac{14}{23} = \frac{19}{1} \times \frac{23}{14} = \frac{437}{14} = 31\frac{3}{14}$, Ans.

23. $167 \div \frac{15}{18} = \frac{167}{1} \times \frac{18}{15} = \frac{3006}{15} = 200\frac{2}{5}$, Ans.

24. $49 \div \frac{16}{29} = \frac{49}{1} \times \frac{29}{16} = \frac{1421}{16} = 88\frac{13}{16}$, Ans.

25. $15 \div \frac{1}{15} = \frac{15}{1} \times \frac{15}{1} = 225$, Ans.

26. $\frac{36}{51} \div \frac{3}{17} = \frac{12}{3} = 4$, Ans.

27. $\frac{31}{97} \div \frac{31}{301} = \frac{\cancel{31}}{97} \times \frac{301}{\cancel{31}} = \frac{301}{97} = 3\frac{10}{97}$, Ans.

28. $\frac{5}{6} \div \frac{8}{9} = \frac{5}{6} \times \frac{9}{8} = \frac{45}{48} = \frac{15}{16}$, Ans.

29. $\frac{11}{15} \div \frac{7}{11} = \frac{11}{15} \times \frac{11}{7} = \frac{121}{105} = 1\frac{16}{105}$, Ans.

30. $\frac{8}{25} \div \frac{14}{17} = \frac{\overset{4}{\cancel{8}}}{25} \times \frac{17}{\underset{7}{\cancel{14}}} = \frac{68}{175}$, Ans.

31. $\frac{16}{21} \div \frac{1}{15} = \frac{16}{21} \times \frac{15}{1} = \frac{240}{21} = 11\frac{3}{7}$, Ans.

32. $\frac{9}{25} \div 7\frac{3}{4} = \frac{9}{25} \times \frac{4}{31} = \frac{36}{775}$, Ans.

33. $\frac{8}{11} \div \frac{164}{9} = \frac{8}{11} \times \frac{9}{148} = \frac{72}{1628} = \frac{18}{407}$, Ans.

34. $11\frac{3}{7} \div \frac{4}{7} = \frac{\overset{20}{\cancel{80}}}{\cancel{7}} \times \frac{\cancel{7}}{\underset{1}{\cancel{4}}} = 20$, Ans.

35. $21\frac{4}{5} \div 18\frac{4}{7} = \frac{109}{5} \times \frac{7}{130} = \frac{763}{650} = 1\frac{113}{650}$, Ans.

36. $17\frac{3}{11} \div 28\frac{11}{26} = \frac{190}{11} \times \frac{26}{739} = \frac{4940}{8129}$, Ans.

37. $161\frac{3}{17} \div 14\frac{3}{5} = \frac{2740}{17} \times \frac{5}{73} = \frac{13700}{1241} = 11\frac{49}{1241}$, Ans.

38. $\frac{7}{11} \times \frac{4}{5} = \frac{28}{55}; \frac{3}{5} \times \frac{8}{11} = \frac{24}{55}; \frac{\overset{7}{\cancel{28}}}{\cancel{55}} \times \frac{\cancel{55}}{\underset{6}{\cancel{24}}} = \frac{7}{6} = 1\frac{1}{6}$, Ans.

39. $\frac{5}{9} \times 7\frac{2}{11} = \frac{400}{99}; \frac{4}{11} \times 17\frac{3}{4} = \frac{488}{77}; \frac{\overset{100}{\cancel{400}}}{\underset{9}{\cancel{99}}} \times \frac{\overset{7}{\cancel{77}}}{\underset{122}{\cancel{488}}} = \frac{700}{1098}$

$\qquad\qquad\qquad\qquad\qquad\qquad [= \frac{350}{549}$, Ans.

40. $\frac{6}{17} \times 15 = \frac{90}{17}; \frac{7}{15} \times 22 = \frac{154}{15}; \frac{90}{17} \times \frac{15}{154} = \frac{1350}{2618} =$

$\qquad\qquad\qquad\qquad\qquad\qquad [= \frac{675}{1309}$, Ans.

41. $\frac{7}{7} - \frac{5}{7} = \frac{2}{7}; \frac{2}{3} \times \frac{2}{7} = \frac{4}{21}; \frac{2}{7} - \frac{4}{21} = \frac{2}{21}; \frac{2}{21} \times 3675 =$

$\qquad\qquad\qquad\qquad\qquad [350; \div 7 = \$50$, Ans.

42. $1 - \frac{1}{3} = \frac{2}{3}; \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}; \frac{1}{3} + \frac{4}{9} = \frac{7}{9}; 1 - \frac{7}{9} = \frac{2}{9}; \frac{2}{9} \times$

$\qquad\qquad\qquad [\frac{1}{7} = \frac{2}{63}; \frac{2}{63} \times 5000 = \$158\frac{46}{63}$, Ans.

(ART. 240, p. 176.)

2. $\dfrac{9)29\frac{3}{8}}{3, \text{rem. } 2\frac{3}{8}}; 2\frac{3}{8} = \frac{19}{8}; \frac{19}{8} \times 9 = \frac{19}{72}; 3 + \frac{19}{72} = 3\frac{19}{72}$, Ans.

3. $\dfrac{7)14\frac{1}{2}}{2, \text{rem. } \frac{1}{2}}; \frac{1}{2} \times 7 = \frac{1}{14}; 2 + \frac{1}{14} = 2\frac{1}{14}$, Ans.

4. $\dfrac{8)13\frac{3}{8}}{1, \text{rem. } 5\frac{3}{8}}; 5\frac{3}{8} = \frac{43}{8}; \frac{43}{8} \times 8 = \frac{43}{64}; 1 + \frac{43}{64} = 1\frac{43}{64}$, Ans.

5. $\dfrac{6)14\frac{2}{5}}{2}$, rem. $2\frac{2}{5}$; $2\frac{2}{5}=\frac{12}{5}$; $\frac{12}{5}\times 6=\frac{12}{30}$; $2+\frac{12}{30}=2\frac{12}{30}$, Ans.

6. $\dfrac{9)37\frac{7}{7}}{4}$, rem. 1., $1\frac{2}{7}=\frac{12}{7}$; $\frac{12}{7}\times 9=\frac{12}{63}$; $4+\frac{12}{63}=4\frac{12}{63}$, Ans.

7. $\dfrac{11)96\frac{2}{3}}{8}$, rem. $8\frac{2}{3}$; $8\frac{2}{3}=\frac{26}{3}$; $\frac{26}{3}\times 11=\frac{26}{33}$; $8+\frac{26}{33}=\$8\frac{26}{33}$,
[Ans.

8. $\dfrac{8)167\frac{7}{11}}{20}$, rem. $7\frac{7}{11}$; $7\frac{7}{11}=\frac{84}{11}$; $\frac{84}{11}\times 8=\frac{84}{88}=\frac{21}{22}$; $20+\frac{21}{22}$
[$=20\frac{21}{22}$cwt., Ans.

9. $\frac{7}{8}\times\frac{1723}{1}=\frac{12061}{8}=1507\frac{5}{8}$; $\dfrac{12)1507\frac{5}{8}}{125,\text{rem.}7\frac{5}{8}}$; $7\frac{5}{8}=\frac{61}{8}$;
[$\frac{61}{8}\times 12=\frac{61}{96}$; $\$125+\frac{61}{96}=\$125\frac{61}{96}$, Ans.

10. $\dfrac{19)1107\frac{3}{8}}{5}$, rem. $12\frac{3}{8}$; $12\frac{3}{8}=\frac{99}{8}$; $\frac{99}{8}\times 19=\frac{99}{152}$; $5+\frac{99}{152}=$
$5\frac{99}{152}$; $\frac{99}{152}\times 100=\frac{9900}{152}=.65\frac{8}{38}$; $\$5+.65\frac{8}{38}=$
[$\$5.65\frac{8}{38}$, Ans.

11. $\frac{2}{7}$ of $\$1=\frac{2\times 100}{7}=\frac{200}{7}$; $\frac{200}{7}=.28\frac{4}{7}$; $\$3+.28\frac{4}{7}=3.28\frac{4}{7}$;
$\dfrac{15)3.28\frac{4}{7}}{\$0.21,\text{rem.}13\frac{4}{7}}$; $13\frac{4}{7}=\frac{95}{7}$; $\frac{95}{7}\times 15=\frac{95}{105}=\frac{19}{21}$;
[$\$0.21+\frac{19}{21}=\$0.21\frac{19}{21}$, Ans.

12. $\dfrac{12)\$3.75\frac{3}{4}}{\$.31,\text{rem.}3\frac{3}{4}}$; $3\frac{3}{4}=\frac{15}{4}$; $\frac{15}{4}\times 12=\frac{15}{48}=\frac{5}{16}$;
[$\$0.31+\frac{5}{16}=\$0.31\frac{5}{16}$, Ans.

13. $\dfrac{19)375\frac{11}{16}}{19,\text{rem.}14\frac{11}{16}}$; $14\frac{11}{16}=\frac{235}{16}$; $\frac{235}{16}\times 19=\frac{235}{304}$;
[$19+\frac{235}{304}=19\frac{235}{304}$ acres, Ans.

REDUCTION OF COMPLEX FRACTIONS.

(ART. 242, p. 178.)

4. $\dfrac{4\frac{2}{7}}{\frac{2}{3}}=\dfrac{\frac{30}{7}}{\frac{2}{3}}=\frac{30}{7}\times\frac{3}{2}=\frac{90}{14}=6\frac{6}{14}$, Ans.

5. $\dfrac{\frac{3}{4}}{5\frac{2}{3}} = \dfrac{\frac{3}{4}}{\frac{17}{3}} = \frac{3}{4} \times \frac{3}{17} = \frac{9}{68}$, Ans.

6. $\dfrac{7}{4\frac{2}{3}} = \dfrac{7}{\frac{14}{3}} = 7 \times \frac{3}{14} = \frac{21}{14} = 1\frac{1}{2}$, Ans.

7. $\dfrac{7\frac{4}{11}}{8} = \dfrac{\frac{81}{11}}{\frac{8}{1}} = \frac{81}{11} \times \frac{1}{8} = \frac{81}{88}$, Ans.

8. $\dfrac{6\frac{2}{9}}{8\frac{2}{3}} = \dfrac{\frac{56}{9}}{\frac{26}{3}} = \frac{56}{9} \times \frac{3}{26} = \frac{168}{234} = \frac{28}{39}$, Ans.

9. $\dfrac{\frac{2}{7}}{\frac{3}{5}} = \frac{2}{7} \times \frac{5}{3} = \frac{10}{21}$, Ans.

10. $\dfrac{8}{\frac{1}{3}} = \dfrac{\frac{8}{1}}{\frac{1}{3}} = \frac{8}{1} \times \frac{3}{1} = 24$, Ans.

11. $\dfrac{\frac{4}{7}}{2} = \dfrac{\frac{4}{7}}{\frac{2}{1}} = \frac{4}{7} \times \frac{1}{2} = \frac{4}{14} = \frac{2}{7}$, Ans.

12. $\dfrac{5\frac{1}{3}}{\frac{3}{7}} = \dfrac{\frac{16}{3}}{\frac{3}{7}} = \frac{16}{3} \times \frac{7}{3} = \frac{112}{9} = 12\frac{4}{9}$, Ans.

13. $\dfrac{\frac{1}{4}}{6\frac{1}{2}} = \dfrac{\frac{1}{4}}{\frac{13}{2}} = \frac{1}{4} \times \frac{2}{13} = \frac{2}{52} = \frac{1}{26}$, Ans.

14. $\dfrac{3}{2\frac{1}{2}} = \dfrac{\frac{3}{1}}{\frac{5}{2}} = \frac{3}{1} \times \frac{2}{5} = \frac{6}{5} = 1\frac{1}{5}$, Ans.

15. $\dfrac{3\frac{1}{4}}{9} = \dfrac{\frac{13}{4}}{\frac{9}{1}} = \frac{13}{4} \times \frac{1}{9} = \frac{13}{36}$, Ans.

16. $\dfrac{11\frac{2}{3}}{12\frac{3}{5}} = \dfrac{\frac{35}{3}}{\frac{63}{5}} = \frac{35}{3} \times \frac{5}{63} = \frac{175}{189} = \frac{25}{27}$, Ans.

17. $\dfrac{\dfrac{7\frac{7}{9}}{11\frac{4}{5}}}{7} = \dfrac{\dfrac{70}{9}}{\dfrac{59}{5}} = \frac{70}{9} \times \frac{5}{59} \times \frac{1}{7} = \frac{350}{3717} = \frac{50}{531}$, Ans.

18. $\dfrac{\frac{7}{9}}{\frac{3}{5}} = \frac{7}{9} \times \frac{5}{3} \times \frac{5}{1} = \frac{175}{27} = 6\frac{13}{27}$, Ans.

(Art. 243, p. 179.)

1. $28\frac{36}{42}=28\frac{27}{30}$; $\frac{27}{30}=\frac{27}{7}\times\frac{7}{30}=\frac{9}{10}$; $28+\frac{9}{10}=28\frac{9}{10}=$

$\frac{289}{10}$; $\frac{1}{3}\times\frac{2}{7}\times\frac{289}{10}=\frac{578}{210}=2\frac{158}{210}$; $3\frac{39\frac{1}{2}}{105}=3\frac{\frac{79}{2}}{105}$;

$\frac{\frac{79}{2}}{105}=\frac{79}{2}\times\frac{1}{105}=\frac{79}{210}$; $3+\frac{79}{210}=3\frac{79}{210}$; $2\frac{158}{210}+$

$[3\frac{79}{210}=6\frac{9}{70}$, Ans.

2. $\frac{1}{9}=\frac{1}{9}$; $2\frac{5}{8}=\frac{21}{8}$; $\frac{45}{94\frac{7}{11}}=\frac{45}{\frac{1041}{11}}=\frac{45}{1}\times\frac{11}{1041}=\frac{495}{1041}$;

$\frac{47\frac{5}{9}}{314\frac{3}{5}}=\frac{\frac{428}{9}}{\frac{1573}{5}}=\frac{428}{9}\times\frac{5}{1573}=\frac{2140}{14157}$; $\frac{1}{9}+\frac{21}{8}+$

$\frac{495}{1041}+\frac{2140}{14157}$.   8)9 8 1041 14157

                                    8   347   4719

$3\times 8\times 347\times 4719=39299832.$

```
   9│39299832
   8│ 4366648 ×    1 =   4366648
1041│ 4912479 ×   21 = 103162059
14157│  37752 ×  495 =  18687240
           2776 × 2140 =  5940640
```

$$\frac{132156587}{39299832}=3\frac{14257091}{39299832},\ \text{Ans.}$$

3. $\frac{49\frac{5}{8}}{97}=\frac{\frac{397}{8}}{97}=\frac{397}{8}\times\frac{1}{97}=\frac{397}{776}$; $\frac{34\frac{3}{5}}{145\frac{3}{11}}=\frac{\frac{173}{5}}{\frac{1598}{11}}=\frac{173}{5}\times$

$[\frac{11}{1598}=\frac{1903}{7990}$; $\frac{397}{776}-\frac{1903}{7990}=\frac{847651}{3100120}$, Ans.

4. $\dfrac{27}{37\frac{4}{5}} = \dfrac{\frac{27}{1}}{\frac{189}{5}} = \dfrac{\overset{1}{\cancel{27}}}{1} \times \dfrac{5}{\underset{7}{\cancel{189}}} = \dfrac{5}{7}$;

$\dfrac{87\frac{2}{9}}{98\frac{1}{8}} = \dfrac{\frac{785}{9}}{\frac{785}{8}} = \dfrac{\cancel{785}}{9} \times \dfrac{8}{\cancel{785}} = \dfrac{8}{9}$;

$\dfrac{\frac{7}{8}}{2\frac{1}{3}} = \dfrac{\frac{7}{8}}{\frac{7}{3}} = \dfrac{\cancel{7}}{8} \times \dfrac{3}{\cancel{7}} = \dfrac{3}{8}$;

$\dfrac{81\frac{5}{11}}{128} = \dfrac{\frac{896}{11}}{\frac{128}{1}} = \dfrac{\overset{7}{\cancel{896}}}{11} \times \dfrac{1}{\cancel{128}} = \dfrac{7}{11}$;

$\dfrac{5}{\cancel{7}} \times \dfrac{\cancel{8}}{\underset{3}{\cancel{9}}} \times \dfrac{\cancel{3}}{\cancel{8}} \times \dfrac{\cancel{7}}{11} = \dfrac{5}{33}$, Ans.

5. $\dfrac{2}{3} \times 7\frac{3}{4} = \dfrac{2}{3} \times \dfrac{31}{4} = \dfrac{62}{12} = \dfrac{31}{6}$; $\dfrac{4}{5} \times 11\frac{4}{11} = \dfrac{4}{5} \times \dfrac{125}{11} =$

$[\dfrac{500}{55} = \dfrac{100}{11}$, $\dfrac{31}{6} \times \dfrac{11}{100} = \dfrac{341}{600}$, Ans.

6. $\dfrac{4}{9} \times \dfrac{91}{1} = \dfrac{364}{9}$; $\dfrac{9}{10} \times \dfrac{87}{1} = \dfrac{783}{10}$; $\dfrac{364}{9} \times \dfrac{10}{783} = \dfrac{3640}{7047}$,
$\hfill$[Ans.

MISCELLANEOUS EXAMPLES IN MULTIPLICATION AND DIVISION OF FRACTIONS.

(PAGE 179.)

1. $2\frac{3}{7} = \dfrac{17}{7}$; $7\frac{4}{17} = \dfrac{123}{17}$; $\dfrac{123}{\cancel{17}} \times \dfrac{\cancel{17}}{7} = \dfrac{123}{7} = 17\frac{4}{7}$, Ans.

2. $8\frac{1}{5} = \dfrac{41}{5}$; $4.68\frac{4}{7} = \dfrac{3280}{7}$; $\dfrac{3280}{7} \times \dfrac{5}{41} = \dfrac{16400}{287}$; $\dfrac{16400}{287}$
$\hfill$[$= \$ 0.57\frac{1}{2}$, Ans.

3. $96\frac{3}{8} = \dfrac{771}{8}$; $1 - \dfrac{5}{8} = \dfrac{8}{8} - \dfrac{5}{8} = \dfrac{3}{8}$; $\dfrac{771}{8} \times \dfrac{3}{8} = \dfrac{2313}{64}$;

$\dfrac{2313}{64} \div 2 = \dfrac{2313}{64} \times \dfrac{1}{2} - \dfrac{2313}{128} = 18\frac{9}{128}$yd.; Ans.

7

4. $18\frac{1}{3} \times \frac{3}{4} = \frac{55}{3} \times \frac{3}{4} = \frac{165}{12}$; $6\frac{1}{2} \times \frac{2}{3} = \frac{13}{2} \times \frac{2}{3} = \frac{26}{6}$; $\frac{165}{12}$

$[\div \frac{26}{6} = \frac{165}{\overset{6}{12}} \times \frac{6}{26} = \frac{165}{52} = 3\frac{9}{52}$, Ans.

5. $1\frac{1}{3} = 1 + \frac{1}{3}$; $\frac{1}{3} = \frac{1}{4} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$; $1 + \frac{1}{6} = 1\frac{1}{6} = \frac{7}{6}$; $1 \div$

$[\frac{7}{6} = \frac{1}{1} \times \frac{6}{7} = \frac{6}{7}$, Ans.

6. $1 + \frac{11}{12} = \frac{12}{12} + \frac{11}{12} = \frac{23}{12}$; $250 \times 12 = 3000$; $3000 \div$

$[23 = \$130\frac{10}{23}$, horse.

$250 \times 11 = 2750$; $2750 \div 23 = \$119\frac{13}{23}$, chaise.

$\frac{7}{11} \times 130\frac{10}{23} = \frac{7}{11} \times \frac{3000}{23} = \frac{21000}{253} = \$83\frac{1}{253}$, harness.

7. $\$200 \times \frac{3}{4} = \$150$; $\$150 - \$144 = \$6$; $\$6 \times 4 =$

$[\$24.00$, Ans.

8. $\frac{1}{5} \times \frac{3}{4} \times 3\frac{1}{8} = \frac{1}{5} \times \frac{3}{4} \times \frac{25}{8} = \frac{75}{160} = \frac{25}{32}$; $\frac{25}{32} \times \frac{1}{2} = \frac{25}{64}$;

$\frac{25}{32} \times \frac{25}{64} = \frac{625}{2048}$; $\frac{625}{2048} \div \frac{1}{2} = \frac{625}{2048} \times \frac{2}{1} = \frac{1250}{2048} =$

$[\frac{625}{1024}$, Ans.

9. $13\frac{7}{8} \times 25\frac{3}{4} = \frac{111}{8} \times \frac{103}{4} = \frac{11433}{32}$; $\frac{11433}{32} \div 2\frac{3}{5} = \frac{11433}{32}$

$[\div \frac{13}{5}; \frac{11433}{32} \times \frac{5}{13} = \frac{57165}{416} = 137\frac{173}{416}$ bushels, Ans.

10. $3\frac{1}{2} \times 9\frac{7}{8} = \frac{7}{2} \times \frac{79}{8} = \frac{553}{16}$; $\frac{553}{1} \div \frac{553}{16} = \frac{553}{1} \times \frac{16}{553} =$

$[16$ days, Ans.

11. $\$0.90 \div 4 = \$0.22\frac{1}{2}$; that is, he will receive $\frac{1}{2}$ more.

12. $9\frac{4}{5} \times 16\frac{3}{4} = \frac{49}{5} \times \frac{67}{4} = \frac{3283}{20} = 164\frac{3}{20}$ miles per day.

$164\frac{3}{20} \times 4 = 656\frac{3}{5}$ miles in 4 days.

$164\frac{3}{20} \times 10 = 1641\frac{1}{2}$ miles in 10 days.

24h. $- 16\frac{3}{4}$h. $= 7\frac{1}{4}$h.; $7\frac{1}{4} \times 2 = 14\frac{1}{2}$ miles to deduct for drift in the night.

$164\frac{3}{20} - 14\frac{1}{2} = 149\frac{13}{20}$ miles gained per day.

$149\frac{13}{28} \times 10 = 1496\frac{1}{2}$ miles gained in 10 days.

$656\frac{3}{5}$m. $+ 1496\frac{1}{2}$m. $= 2153\frac{1}{10}$ miles' distance sailed in 14 days.

$2317\frac{1}{4}$m. $- 2153\frac{1}{10}$m. $= 164\frac{3}{20}$ miles remaining, which distance will be sailed the last day; therefore the boat will arrive at her port in 15 days, Ans.

A PROPOSED NUMERATOR OR DENOMINATOR.
(ART. 244, p. 181.)

3. $34 \div 17 = 2; \dfrac{17 \times 2}{20 \times 2} = \dfrac{34}{40}$, Ans.

4. $3\frac{6}{7} = \frac{27}{7}; 9 \div 27 = \frac{1}{3}; \dfrac{27 \times \frac{1}{3}}{7 \times \frac{1}{3}} = \dfrac{9}{2\frac{1}{3}}$, Ans.

5. $5 \div 4 = 1\frac{1}{4}; \dfrac{4 \times 1\frac{1}{4}}{1 \times 1\frac{1}{4}} = \dfrac{5}{1\frac{1}{4}}$, Ans.

6. $12 \div 16 = \frac{3}{4}; \dfrac{15 \times \frac{3}{4}}{16 \times \frac{3}{4}} = \dfrac{11\frac{1}{4}}{12}$, Ans.

7. $15 \div 5 = 3; \dfrac{3 \times 3}{5 \times 3} = \dfrac{9}{15}$, Ans.

8. $2 \div 3 = \frac{2}{3}; \dfrac{7 \times \frac{2}{3}}{3 \times \frac{2}{3}} = \dfrac{4\frac{2}{3}}{2}$, Ans.

9. $35 \div 20 = 1\frac{3}{4}; \dfrac{19 \times 1\frac{3}{4}}{20 \times 1\frac{3}{4}} = \dfrac{33\frac{1}{4}}{35}$, Ans.

10. $\frac{12}{17} = \frac{1}{3}; \dfrac{1 \times 4}{3 \times 4} = \dfrac{4}{12}; \frac{67}{201} = \frac{1}{3}; \dfrac{1 \times 4}{3 \times 4} = \dfrac{4}{12}$, Ans.

COMMON NUMERATOR.
(ART. 246, p. 182.)

2. 24, the least common multiple of the numerators, 8, 4, 3. 6.

$$
\begin{array}{l}
\underline{24} \\
8 \mid 3 \times 9 = 27 \text{ denominator for } \frac{8}{9} = \frac{24}{27} \\
4 \mid 6 \times 7 = 42 \quad \text{``} \quad \text{``} \quad \frac{4}{6} = \frac{24}{42} \\
3 \mid 8 \times 4 = 32 \quad \text{``} \quad \text{``} \quad \frac{3}{4} = \frac{24}{32} \\
6 \mid 4 \times 7 = 28 \quad \text{``} \quad \text{``} \quad \frac{6}{7} = \frac{24}{28}
\end{array} \Bigg\} \text{Ans.}
$$

3. $\frac{5}{8}, 2\frac{1}{2}, 1\frac{3}{7} = \frac{5}{8}, \frac{5}{2}, \frac{10}{7}.$

   10, the least common multiple of the numerators, 5, 5, and 10.

$$
\begin{array}{r|l}
 & 10 \\ \hline
5 & 2 \times 8 = 16 \text{ denominator for } \frac{5}{8} = \frac{10}{16} \\
5 & 2 \times 2 = 4 \qquad\text{``}\qquad\text{``}\quad \frac{5}{2} = \frac{10}{4} \\
10 & 1 \times 7 = 7 \qquad\text{``}\qquad\text{``}\quad \frac{10}{7} = \frac{10}{7}
\end{array} \Bigg\} \text{ Ans.}
$$

4. $4\frac{4}{15}, 6\frac{2}{5}, 6\frac{2}{3} = \frac{64}{15}, \frac{32}{5}, \frac{20}{3}.$

   320 is the least common multiple of 64, 32, 20.

$$
\begin{array}{r|l}
 & 320 \\ \hline
64 & 5 \times 15 = 75 \text{ denominator of } \frac{64}{15} = \frac{320}{75} \\
32 & 10 \times 5 = 50 \qquad\text{``}\qquad\text{``}\quad \frac{32}{5} = \frac{320}{50} \\
20 & 16 \times 3 = 48 \qquad\text{``}\qquad\text{``}\quad \frac{20}{3} = \frac{320}{48}
\end{array} \Bigg\} \text{ Ans.}
$$

From the above process, it is evident they will meet in 320 days. A will have gone round the island 75 times, and have travelled $75 \times 50 = 3750$ miles. B will have gone round the island 50 times, and have travelled $50 \times 50 = 2500$ miles. C will have gone round the island 48 times, and have travelled $48 \times 50 = 2400$ miles.

GREATEST COMMON DIVISOR OF FRACTIONS.

(Art. 248, p. 183.)

2. Greatest common divisor of the numerators $= 4$ } Ans.
   Least common denominator of the fractions $= \overline{315}$

3. $12\frac{3}{8}, 9\frac{3}{4}, 8\frac{1}{4} = \frac{99}{8}, \frac{39}{4}, \frac{33}{4}.$
   Greatest common divisor of the numerators $= 3$ } Ans.
   Least common denominator of the fractions $= \overline{8}$

Note.— $\frac{3}{8}$ will divide each of the fractions, $12\frac{3}{8}$, $9\frac{3}{4}$, and $8\frac{1}{4}$, without a remainder.

4. Greatest common divisor of the numerators $= 1$ } Ans.
   Least common denominator of the fractions $= \overline{60}$

5. $3\frac{4}{5}$, $5\frac{7}{10}$, $2\frac{8}{15} = \frac{19}{5}, \frac{57}{10}, \frac{38}{15}$.
Greatest common divisor of the numerators $= 19$ }
Least common denominator of the fractions $= \overline{30}$ } Ans.

6. $33\frac{3}{4}$, $67\frac{1}{2}$, $70\frac{7}{8} = \frac{135}{4}, \frac{135}{2}, \frac{567}{8}$.
Greatest common divisor of the numerators $= 27$ }
Least common denominator of the fractions $= \overline{8} = 3\frac{3}{8}$ } Ans.
$33\frac{3}{4} \div 3\frac{3}{8} = 10$ bags; $67\frac{1}{2} \div 3\frac{3}{8} = 20$ bags; $70\frac{7}{8} \div 3\frac{3}{8} =$
21 bags. $10 + 20 + 21 = 51$ bags, Ans.

7. $73\frac{7}{11}$, $88\frac{4}{11}$, $139\frac{10}{11} = \frac{810}{11}, \frac{972}{11}, \frac{1539}{11}$.
Greatest common divisor of numerators $= 81$
Least common denominator of the fractions $= \overline{11}$
$\frac{810}{11} + \frac{972}{11} + \frac{1539}{11} = \frac{3321}{11}$; $\frac{3321}{11} \div 8\frac{1}{11} = 41$ lots.
$81 \div 11 = 7\frac{4}{11}$ acres, the size of each lot.

LEAST COMMON MULTIPLE OF FRACTIONS.

(ART. 250, p. 184.)

2. $\frac{4}{5}$, $\frac{8}{3}$, and $\frac{6}{7}$.
Least common mult. of the numerators $= 24$
Greatest com. div. of the denominators $= \overline{1} = 24$, Ans.

3. $3\frac{15}{16}$, $7\frac{7}{8}$, $5\frac{1}{4} = \frac{63}{16}, \frac{63}{8}, \frac{21}{4}$.
Least common mult. of the numerators $= 63$
Greatest com. div. of the denominators $= \overline{4} = 15\frac{3}{4}$, Ans.

4. $\frac{3}{8}$, $\frac{6}{7}$, $\frac{9}{10}$.
Least common mult. of the numerators $= 18$
Greatest com. div. of the denominators $= \overline{1} = 18$, Ans.

NOTE. — By this operation we find that 18 is the least whole number that can be divided by the fractions $\frac{3}{8}$, $\frac{6}{7}$, and $\frac{9}{10}$, without a remainder.

5. $2\frac{1}{4}$, $4\frac{1}{2}$, $9\frac{3}{8} = \frac{9}{4}, \frac{9}{2}, \frac{75}{8}$.
Least common mult. of the numerators $= 225$
Greatest com. div. of the denominators $= \overline{2} = \$112\frac{1}{2}$.
$\frac{225}{2} \div \frac{9}{4} = 50$ sheep; $\frac{225}{2} \div \frac{9}{2} = 25$ calves; $\frac{225}{2} \div \frac{75}{8}$
$= 12$ yearlings.

7*

6. $3\frac{1}{2}, 4\frac{2}{3}, 5\frac{1}{4} = \frac{7}{2}, \frac{14}{3}, \frac{21}{4}$.

   $80 \div \frac{7}{2} = 22\frac{6}{7}$ hours, the time A will go round the island. $80 \div \frac{14}{3} = 17\frac{1}{7}$ hours, the time B will go round the island. $80 \div \frac{21}{4} = 15\frac{5}{21}$ hours, the time C will go round the island.

   $22\frac{6}{7}, 17\frac{1}{7}, 15\frac{5}{21} = \frac{160}{7}, \frac{120}{7}, \frac{320}{21}$.

   Least common mult. of $160, 120, 320 = 960$

   Greatest common divisor of $7, 7, 21 = \dfrac{960}{7} = 137\frac{1}{7}$ hours.

In $137\frac{1}{7} \div 8 = 17\frac{1}{7}$ days, they will all meet at the point from which they started.

$\frac{960}{7} \div \frac{160}{7} = 6$ times, A will go round the island.

$\frac{960}{7} \div \frac{120}{7} = 8$ times, B will go round.

$\frac{960}{7} \div \frac{320}{21} = 9$ times, C will go round.

$80 \times 6 = 480$ miles, A travels. $80 \times 8 = 640$ miles, B travels. $80 \times 9 = 720$ miles, C travels.

7. $3\frac{1}{2}, 4\frac{2}{3}, 5\frac{1}{4} = \frac{7}{2}, \frac{35}{3}, \frac{21}{4}$.

   Least common multiple of $7, 35, 21 = 105$

   Greatest common divisor of $2, 8, 4 = \dfrac{105}{2} = 52\frac{1}{2}$.

As $52\frac{1}{2}$ can be divided by each of the fractions $3\frac{1}{2}, 4\frac{2}{3}, 5\frac{1}{4}$, it is certain that twice $52\frac{1}{2} = 105$ may be divided by them; and no less whole number than this is a multiple of $52\frac{1}{2}$.

REDUCTION OF DENOMINATE FRACTIONS.

(Art. 253, p. 185.)

2. $\frac{1}{1200} \times \frac{20}{1} \times \frac{12}{4} \times \frac{3}{1} = \frac{360}{1200} = \frac{3}{10}$, Ans.

3. $\frac{1}{560} \times \frac{12}{1} \times \frac{20}{1} \times \frac{24}{1} = \frac{576}{560} = \frac{3}{5}$, Ans.

4. $\frac{5}{2592} \times \frac{12}{1} \times \frac{9}{1} \times \frac{3}{1} = \frac{1440}{2592} = \frac{5}{9}$, Ans.

5. $\frac{3}{8800} \times \frac{4}{1} \times \frac{25}{1} \times \frac{16}{1} = \frac{4800}{8800} = \frac{6}{11}$, Ans.

6. $\frac{3}{8000} \times \frac{20}{1} \times \frac{4}{1} \times \frac{25}{1} = \frac{6000}{8000} = \frac{3}{4}$, Ans.

7. $\dfrac{1}{225} \times \dfrac{5}{1} \times \dfrac{4}{1} \times \dfrac{2\frac{1}{4}}{1} = \dfrac{45}{225} = \dfrac{1}{5}$, Ans.

8. $\dfrac{1}{110880} \times \dfrac{8}{1} \times \dfrac{40}{1} \times \dfrac{16\frac{1}{2}}{1} \times \dfrac{12}{1} = \dfrac{63360}{110880} = \dfrac{4}{7}$, Ans.

9. $\frac{1}{38016} \times \frac{3}{1} \times \frac{8}{1} \times \frac{40}{1} \times \frac{16\frac{1}{2}}{1} \times \frac{12}{1} = \frac{570240}{1140480} = \frac{1}{2}$, Ans.

10. $\frac{3}{25090560} \times \frac{4}{1} \times \frac{40}{1} \times \frac{272\frac{1}{4}}{1} \times \frac{144}{1} = \frac{18817920}{25090560} = \frac{3}{4}$,
[Ans.

11. $\frac{1}{1152} \times \frac{4}{1} \times \frac{63}{1} \times \frac{4}{1} = \frac{1008}{1152} = \frac{7}{8}$, Ans.

12. $\frac{3}{320} \times \frac{4}{1} \times \frac{8}{1} \times \frac{2}{1} = \frac{192}{320} = \frac{3}{5}$, Ans.

13. $\frac{1}{4207680} \times \frac{365\frac{1}{4}}{1} \times \frac{24}{1} \times \frac{60}{1} = \frac{525960}{4207680} = \frac{1}{8}$, Ans.

14. $\frac{3}{3200} \times \frac{4}{1} \times \frac{25}{1} \times \frac{16}{1} = \frac{4800}{3200} = \frac{3}{2}$, Ans.

(ART. 254, p. 186.)

2. $\frac{4}{5} \times \frac{1}{4} \times \frac{1}{12} \times \frac{1}{20} = \frac{1}{1200}$, Ans.

3. $\frac{3}{8} \times \frac{1}{24} \times \frac{1}{20} \times \frac{1}{12} = \frac{3}{28800} = \frac{1}{9600}$, Ans.

4. $\frac{5}{9} \times \frac{1}{3} \times \frac{1}{8} \times \frac{1}{12} = \frac{5}{2592}$, Ans.

5. $\frac{6}{11} \times \frac{1}{16} \times \frac{1}{25} \times \frac{1}{4} = \frac{6}{17600} = \frac{3}{8800}$, Ans.

6. $\frac{3}{4} \times \frac{1}{25} \times \frac{1}{4} \times \frac{1}{20} = \frac{3}{8000}$, Ans.

7. $\frac{1}{5} \times \frac{1}{2\frac{1}{4}} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{225}$, Ans.

8. $\frac{4}{7} \times \frac{1}{12} \times \frac{1}{16\frac{1}{2}} \times \frac{1}{40} \times \frac{1}{\underset{2}{8}} = \frac{1}{110880}$, Ans.

9. $\frac{1}{2} \times \frac{1}{12} \times \frac{1}{16\frac{1}{2}} \times \frac{1}{40} \times \frac{1}{8} \times \frac{1}{3} = \frac{1}{38016}$, Ans.

10. $\frac{3}{4} \times \frac{1}{144} \times \frac{1}{272\frac{1}{4}} \times \frac{1}{40} \times \frac{1}{4} = \frac{3}{25090560}$, Ans.

11. $\frac{7}{8} \times \frac{1}{4} \times \frac{1}{\underset{9}{63}} \times \frac{1}{4} = \frac{1}{1152}$, Ans.

12. $\frac{3}{5} \times \frac{1}{2} \times \frac{1}{8} \times \frac{1}{4} = \frac{3}{320}$, Ans.

13. $\frac{1}{8} \times \frac{1}{60} \times \frac{1}{24} \times \frac{1}{365\frac{1}{4}} = \frac{1}{4207680}$, Ans.

14. $\frac{3}{2} \times \frac{1}{16} \times \frac{1}{25} \times \frac{1}{4} = \frac{3}{3200}$, Ans.

(ART. **255,** p. **187.**)

(2.)

s.	d.
1	0
	7

24)7 0

Ans. 3½

(3.)

s.	d.	qr.
28	0	0
		7

9)196 0 0

Ans. 21 9 1½

(4.)

cwt.	qr.	lb.	oz.	dr.
1	0	0	0	0
				7

11)7 0 0 0 0

Ans. 2 13 10 2$\frac{10}{11}$

(5.)

lb.	oz.	dr.
1	0	0
		4

9)4 0 0

Ans. 7 1⅞

(6.)

lb.	oz.	pwt.	gr.
1	0	0	0
			8

9)8 0 0 0

Ans. 10 13 8

(7.)

lb	℥	ʒ	Ɔ	gr.
1	0	0	0	0
				4

13)4 0 0 0 0

Ans. 0 3 5 1 12$\frac{4}{13}$

(8.)

E.E.	qr.	na.	in.
1	0	0	0
			5

9)5 0 0 0

Ans. 2 3 0¼

(9.)

m.	fur.	rd.	ft.	in.
1	0	0	0	0
				11

13)11 0 0 0 0

Ans. 6 30 12 8$\frac{4}{13}$

(10.)

fur.	rd.	ft.	in.
1	0	0	0
			8

9)8 0 0 0

Ans. 35 9 2

(11.)

A.	R.	p.	yd.	ft.	in.
1	0	0	0	0	0
					7

13)7 0 0 0 0 0

Ans. 2 6 4 5 127$\frac{5}{13}$

(12.)

p.	ft.	in.
1	0	0
		9

17)9 0 0

Ans. 144 19$\frac{1}{17}$

(13.)

cord.	ft.	in.
1	0	0
		1

13)1 0 0

Ans. 9 1462$\frac{2}{13}$

(14.)					(15.)		(16.)				
hhd.	gal.	qt.	pt.	gi.	hhd.	gal.	y.	da.	h.	m.	sec.
1	0	0	0	0	1	0	1	0	0	0	0

$$(14.) \quad 1\ 0\ 0\ 0\ 0 \quad \text{—} \quad 2$$

For (14): under the line "2", then $19)\overline{2\ 0\ 0\ 0\ 0}$· — Ans. 6 2 1 0$\frac{4}{19}$

For (15): "·7", then $9)\overline{7\ 0}$ — Ans. 42

For (16): "11", then $23)\overline{11\ 0\ 0\ 0\ 0}$ — Ans. 174 16 26 5$\frac{5}{23}$

17. $\$7\frac{3\frac{3}{11}}{4\frac{2}{5}} = \$7\frac{\frac{36}{11}}{\frac{22}{5}}$; $\frac{36}{11} \times \frac{5}{22} = \frac{180}{242}$; $\$7\frac{180}{242}$, Ans.

(Art. **256**, p. **188**.)

2. $\frac{3 \times 4 + 2}{12 \times 4} = \frac{14}{48} = \frac{7}{24}$, Ans.

3. $21 \times 12 + 9 = 261$; $261 \times 4 + 1 = 1045$; $\left.\begin{array}{r} 1045 \times 3 + 1 \\ 28 \times 12 \times 4 \times 3 \end{array}\right\} = \dfrac{3136}{4032}$
$[= \frac{7}{9}$, Ans.

4. $2 \times 25 + 13 = 63$; $63 \times 16 + 10 = 1018$; $\left.\begin{array}{r} 1018 \times 16 + 2 = 16290 \text{; } 16290 \times 11 + 10 \\ 4 \times 25 \times 16 \times 16 \times 11 \end{array}\right\} = \dfrac{179200}{281600}$
$[= \frac{7}{11}$, Ans.

5. $7 \times 16 + 1 = 113$; $\left.\begin{array}{r} 113 \times 9 + 7 = 1024 \\ 16 \times 16 \times 9 \end{array}\right. = \dfrac{1024}{2304} = \frac{4}{9}$, Ans.

6. $10 \times 20 + 13 = 213$; $\left.\begin{array}{r} 213 \times 24 + 8 = 5120 \\ 12 \times 20 \times 24 \end{array}\right. = \dfrac{5120}{5760} = \frac{8}{9}$, Ans.

7. $\frac{268\frac{4}{5}}{231} = \frac{1344}{1155}$, Ans.

8. $\left.\begin{array}{r} 2 \times 4 = 8 \times 2\frac{1}{4} + 1 = 19 \times 13 + 5 = 252 \\ 4 \times 4 \times 2\frac{1}{4} \times 13 \end{array}\right. = \dfrac{252}{468} = \frac{7}{13}$, Ans.

9. $\left.\begin{array}{r} 2 \times 4 + 3 = 11 \times 2\frac{1}{4} = 24\frac{3}{4} \times 4 + 1 = 100 \\ 5 \times 4 \times 2\frac{1}{4} \times 4 \end{array}\right. = \dfrac{100}{180} = \frac{5}{9}$, Ans.

10. $6 \times 40 + 30 = 270 \times 16\frac{1}{2} + 12 = 4467 \times 12 + 8$
$\left.\begin{array}{r} = 53612 \times 3 = 160836 \times 13 + 12 \\ 8 \times 40 \times 16\frac{1}{2} \times 12 \times 3 \times 13 \end{array}\right\} = \dfrac{2090880}{2471040}$
$[= \frac{11}{13}$, Ans.

11.
$$\begin{aligned}35 \times 16\tfrac{1}{2} + 9 &= 586\tfrac{1}{2} \times 12 + 2 = 7040 \\ 40 \times 16\tfrac{1}{2} \times 12 &= \overline{7920} = \tfrac{8}{9}, \text{ Ans.}\end{aligned}$$

12.
$$\left.\begin{aligned}2 \times 40 + 6 &= 86 \times 30\tfrac{1}{4} + 4 = 2605\tfrac{1}{2} \times 9 + 5 \\ &= 23454\tfrac{1}{2} \times 144 + 127 = 3377575 \times 13 + 5\end{aligned}\right\} = \dfrac{43908480}{}$$
$$4 \times 40 \times 30\tfrac{1}{4} \times 9 \times 144 \times 13 = 81544320$$
$$[= \tfrac{7}{13}, \text{ Ans.}$$

13.
$$\begin{aligned}144 \times 144 + 19 &= 20755 \times 17 + 1 = 352836 \\ 272\tfrac{1}{4} \times 144 \times 17 &= \overline{666468} = \tfrac{9}{17}, \text{ Ans.}\end{aligned}$$

14.
$$\begin{aligned}9 \times 1728 + 1462 &= 17014 \times 13 + 2 = \dfrac{221184}{} \\ 128 \times 1728 \times 13 &= \overline{2875392} = \tfrac{1}{13}, \text{ Ans.}\end{aligned}$$

15.
$$\begin{aligned}6 \times 4 + 2 &= 26 \times 2 + 1 = 53 \times 4 = 212 \times 19 + 4 = 4032 \\ 63 \times 4 \times 2 \times 4 \times 19 &= \overline{38304}\end{aligned}$$
$$[= \tfrac{2}{19}, \text{ Ans.}$$

16.
$$\begin{aligned}\text{1lb. avoirdupois} &= 5760 \text{ grains} \\ \text{1lb. troy} &= \overline{7000} \text{ grains} = 1\tfrac{4}{5}, \text{ Ans.}\end{aligned}$$

17.
$$\begin{aligned}174 \times 24 + 16 = 4192 \times 60 + 26 = 251546 \times 60 + 5 &= 15092765 \times 23 + 5 = 347133600 \\ 365\tfrac{1}{4} \times 24 \times 60 \times 60 \times 23 &= 725824800\end{aligned}$$
$$[= \tfrac{1}{2}, \text{ Ans.}$$

ADDITION OF DENOMINATE FRACTIONS.

(Art. 257, p. 190.)

(2.)

	cwt.	qr.	lb.
$\tfrac{7}{11}$ of a ton =	12	2	$22\tfrac{8}{11}$
$\tfrac{1}{22}$ of a cwt. =		3	$2\tfrac{8}{11}$
Ans.	13	2	0

(3.)

	qr.	na.	in.
$\tfrac{2}{5}$ of a yd. =	2	1	$1\tfrac{7}{20}$
$\tfrac{1}{7}$ of an E. E. =		2	$1\tfrac{1}{14}$
$\tfrac{5}{7}$ of a qr. =		3	$0\tfrac{27}{28}$
Ans.	3	3	$1\tfrac{33}{140}$

(4.)

	fur.	rd.	ft.	in.
$\tfrac{7}{11}$ of a mile =	5	3	10	6
$\tfrac{4}{13}$ of a furlong =		12	5	$0\tfrac{12}{13}$
$\tfrac{9}{22}$ of a yard =			1	$2\tfrac{8}{11}$
	5	15	16	$9\tfrac{93}{143}$ =
Ans.	5	16	0	$3\tfrac{93}{143}$.

(5.)

	A.	R.	p.	ft.	in.
$\frac{4}{5}$ of an A. =	0	2	34	77	113$\frac{1}{4}$
$\frac{2}{3}$ of an A. =	0	2	26	181	72
1$\frac{3}{4}$ of an A. =	0	3	28	155	82$\frac{2}{7}$
	2	1	9	141$\frac{3}{4}$	123$\frac{3}{7}$ =
Ans.	2	1	9	142	87$\frac{3}{7}$

(6.)

	m.	fur.	rd.	ft.	in.
18$\frac{3}{7}$ miles =	18	3	17	2	4$\frac{3}{7}$
23$\frac{11}{14}$ miles =	23	6	11	7	0$\frac{6}{7}$
19$\frac{1}{21}$ miles =	19	0	15	3	11$\frac{1}{7}$
Ans.	61	2	3	13	4$\frac{3}{7}$

(7.)

	gal.	qt.	pt.	gi.
1$\frac{1}{2}$ of a gal. =	0	3	1	1$\frac{1}{3}$
1$\frac{7}{12}$ of a hhd. =	5	1	0	0
Ans.	6	0	1	1$\frac{1}{3}$

(8.)

	d.	h.	m.
$\frac{5}{16}$ of a week =	2	4	30
$\frac{1}{5}$ of a day =		4	48
Ans.	2	9	18

(9.)

	ft.	in.
$\frac{3}{4}$ of a square foot =	0	108
$\frac{1}{2}$ of a foot square =		36
Ans.	1	0

(10.)

rd.	ft.	in.
11	16	5
		6
11	16	11 =
12	0	5

SUBTRACTION OF DENOMINATE FRACTIONS.

(ART. 258, p. 190.)

(2.)

	qr.	na.	in.
$\frac{7}{8}$ E. E. =	4	1	1$\frac{1}{8}$
$\frac{3}{7}$ of a yd. =	1	0	1$\frac{3}{7}$
Ans.	3	0	2$\frac{5}{56}$

(3.)

	fur.	rd.	ft.	in.
$\frac{2}{5}$ of a mile =	1	31	1	10
$\frac{7}{11}$ of a fur. =		25	7	6
	1	5	10$\frac{1}{2}$	4 =
Ans.	1	5	10	10

(4.)

	m.	fur.	rd.	ft.	in.
$\frac{5}{7}$ deg. =	49	3	9	8	$7\frac{4}{7}$
$\frac{3}{5}$ mile =	0	4	32	0	0
Ans.	48	6	17	8	$7\frac{4}{7}$

(5.)

	R.	p.	yd.	ft.	in.
$\frac{4}{11}$ A. =	1	18	5	4	72
$\frac{4}{9}$ rod =	0	0	13	4	0
	1	17	$22\frac{1}{4}$	0	72
			$\frac{1}{4}=2$		36
Ans.	1	17	22	2	108

(6.)

	ft.	in.
$\frac{9}{10}$ cord =	115	$345\frac{3}{5}$
$\frac{2}{11}$ cord =	23	$471\frac{3}{11}$
Ans.	91	$1602\frac{12}{55}$

(7.)

		gal.	qt.	pt.	gi.
$\frac{7}{13}$ hhd.	=	33	3	1	$1\frac{7}{13}$
$\frac{4}{5}$ of $\frac{7}{13}$ hhd. = $\frac{28}{65}$ =		27	0	1	$0\frac{28}{65}$
Ans.		6	3	0	$1\frac{7}{65}$

(8.)

	m.	fur.	rd.	ft.	in.
72m. =	72	0	0	0	0
$\frac{4}{7}$ of 72m. =	41	1	5	·11	$9\frac{3}{7}$
	30	6	34	$4\frac{1}{2}$	$2\frac{4}{7}$
				$\frac{1}{2}=6$	
Ans.	30	6	34	4	$8\frac{4}{7}$

(9.)

	d.	h.	m.	sec.
$\frac{3}{7}$ year =	104	8	34	$17\frac{4}{7}$
$\frac{4}{5}$ week =	3	2	40	0
Ans.	101	5	54	$17\frac{4}{7}$

(10.)

	R.	p.	yd.	ft.	in.
$\frac{4}{11}$ A. =	1	18	5	4	72
$\frac{1}{2}$ ft. =					72
Ans.	1	18	5	4	0

MISCELLANEOUS EXAMPLES IN FRACTIONS.

(Page **190**.)

1. $17\frac{3}{11} = \frac{190}{11}$; $4\frac{7}{9} = \frac{43}{9}$; $\frac{190}{11} \times \frac{43}{9} = \frac{8170}{99} = 82\frac{52}{99}$ miles = 82m. 4fur. 8rd. 1ft. 4in., Ans.

2. $29\frac{7}{13} = \frac{384}{13}$; $\frac{384}{13} \times \frac{384}{13} = \frac{147456}{169} = 872\frac{88}{169}$ poles = 5A. 1R. $32\frac{88}{169}$p.; $\frac{88}{169}$p. = 141ft. $109\frac{155}{169}$in., Ans.

3. $17\frac{3}{4} = \frac{71}{4}$; $7\frac{1}{11} = \frac{78}{11}$; $4\frac{3}{5} = \frac{23}{5}$; $\frac{71}{4} \times \frac{78}{11} \times \frac{23}{5} = \frac{127374}{220} = 578\frac{107}{110}$ feet; $578\frac{107}{110} \div 128 = 4$ cords $66\frac{107}{110}$ cubic feet, Ans.

4. $19\frac{7}{8} = \frac{159}{8}$; $6\frac{3}{4} = \frac{27}{4}$; $\frac{159}{8} \times \frac{27}{4} = \frac{4293}{32} = \$134.15\frac{5}{8}$, Ans.

5. $376\frac{1}{16} = \frac{6172}{16}$; $75\frac{3}{8} = \frac{603}{8}$; $\frac{6172}{16} \times \frac{603}{8} = \frac{4087787}{144} =$ $28387.06\frac{1}{4}$, Ans.

6. $17\frac{19}{112} = \frac{1923}{112}$; $4.75 = \frac{475}{1}$; $\frac{1923}{112} \times \frac{475}{1} = \frac{913425}{112} =$ $81.55\frac{65}{112}$, Ans.

7. $1670\frac{7}{13} = \frac{21717}{13}$; $12\frac{3}{4} = \frac{51}{4}$; $\frac{21717}{13} \times \frac{51}{4} = \frac{1107567}{52} =$ $212.99\frac{19}{52}$, Ans.

8. $28\frac{4}{11} = \frac{312}{11}$; $11\frac{3}{4} = \frac{47}{4}$; $\frac{312}{11} \times \frac{47}{4} = \frac{14684}{44} = $ $333.27\frac{3}{11}$, Ans.

9. $37\frac{11}{63} = \frac{2342}{63}$; $17.62\frac{1}{2} = \frac{3525}{2}$; $\frac{2342}{63} \times \frac{3525}{2} = \frac{8255550}{126}$ $= $ $655.20\frac{5}{21}$, Ans.

10. $\frac{7}{8} \times \frac{575}{1} = \frac{4025}{8} = $ $5.03\frac{1}{8}$, Ans.

11. $139\frac{4}{7} = \frac{977}{7}$; $38\frac{4}{5} = \frac{194}{5}$; $\frac{977}{7} \times \frac{194}{5} = \frac{189538}{35} =$ $5415\frac{13}{35}$ poles $= 33$A. 3R. $15\frac{13}{35}$p., Ans.

12. $11\frac{3}{5} = \frac{58}{5}$; $\frac{58}{5} \times \frac{15}{1} = \frac{870}{5}$; $\frac{870}{5} \times \frac{1}{9} = \frac{870}{45} = 19\frac{1}{3}$ feet, Ans.

13. $18\frac{3}{8} = \frac{147}{8}$; $48.15\frac{3}{4} = \frac{19263}{4}$; $\frac{19263}{4} \times \frac{8}{147} = \frac{154104}{588}$ $= $ $2.62\frac{4}{49}$, Ans.

14. $98\frac{7}{8} = \frac{791}{8}$; $\frac{791}{8} \times \frac{1}{63} = \frac{791}{504} = $ $1.56\frac{119}{126}$, Ans.

15. $8\frac{2}{9} = \frac{74}{9}$; $\frac{74}{9} \times \frac{1}{5} = \frac{74}{45} = $ $1.64\frac{4}{9}$, Ans.

16. $11 \times 63 = 693$; $693 + 17\frac{1}{2} = 710\frac{1}{2} = \frac{1421}{2}$; $19\frac{3}{4} = \frac{79}{4}$; $\frac{1421}{2} \times \frac{79}{4} = \frac{112259}{8} = $ $140.32\frac{3}{8}$, Ans.

17. $63 \times 4 \times 2 = 504$pts.; $1\frac{3}{4} = \frac{7}{4}$; $\frac{504}{1} \times \frac{4}{7} = \frac{2016}{7} = 288$ bottles, Ans.

18. $18\frac{7}{12} = \frac{223}{12}$; $10\frac{5}{12} = \frac{125}{12}$; $7\frac{11}{12} = \frac{95}{12}$; $\frac{223}{12} \times \frac{125}{12} \times \frac{95}{12}$ $= \frac{2648125}{1728} = 1532\frac{829}{1728}$; $1532\frac{829}{1728} \div 128 = 11$ cords, $124\frac{829}{1728}$ cubic feet, Ans.

19. $6\frac{7}{8} = \frac{55}{8}$; $65\frac{3}{4} = \frac{263}{4}$; $\frac{55}{8} \times \frac{263}{4} = \frac{14465}{32} = $ $4.52\frac{1}{32}$, Ans.

20. $8\frac{3}{4} = \frac{35}{4}$; $5\frac{7}{12} = \frac{67}{12}$; $3 = \frac{3}{1}$; $\frac{35}{4} \times \frac{67}{12} \times \frac{3}{1} = \frac{7035}{48} =$ $146\frac{9}{16}$ feet, Ans.

21. $46\frac{5}{12} = \frac{557}{12}$; $17\frac{1}{2} = \frac{35}{2}$; $\frac{557}{12} \times \frac{35}{2} = \frac{19495}{24} = 812\frac{7}{24}$ feet, Ans.

8

22. $2\frac{5}{12} + 3\frac{1}{12} = 5\frac{6}{12}$; $5\frac{6}{12} \times \frac{2}{1} = 11$; $11 \times 5\frac{1}{2} = 60\frac{1}{2}$; $2\frac{5}{12}$
$= \frac{29}{12}$; $3\frac{1}{12} = \frac{37}{12}$; $\frac{29}{12} \times \frac{37}{12} \times \frac{2}{1} = \frac{2146}{144} = 14\frac{95}{144}$; $60\frac{1}{2}$
$+ 14\frac{95}{144} = 75\frac{23}{144}$; $75\frac{23}{144} \times \frac{7}{1} = 527\frac{53}{144}$ feet, Ans.

$5\frac{6}{12} = \frac{66}{12}$; $2\frac{5}{12} = \frac{29}{12}$; $3\frac{1}{12} = \frac{37}{12}$; $\frac{66}{12} \times \frac{29}{12} \times \frac{37}{12} \times \frac{7}{1} =$
$\frac{495726}{1728} = 286\frac{358}{288}$ cubic feet, second Ans.

23. $12 + 11\frac{1}{2} = 23\frac{1}{2}$; $23\frac{1}{2} \times 2 = 47$; $47 \times 7\frac{1}{2} = 352\frac{1}{2}$; 12
$\times 11\frac{1}{2} = 138$; $352\frac{1}{2} + 138 = 490\frac{1}{2}$; $490\frac{1}{2} \times .02\frac{3}{4} =$
$\$13.48\frac{7}{8}$, Ans.

24. $14\frac{1}{2} = \frac{29}{2}$; $10\frac{1}{4} = \frac{41}{4}$; $16\frac{1}{2} = \frac{33}{2}$; $\frac{29}{2} \times \frac{33}{2} = \frac{957}{4} =$
$239\frac{1}{4}$; $\frac{41}{4} \times \frac{33}{2} = \frac{1353}{8} = 169\frac{1}{8}$; $239\frac{1}{4} + 169\frac{1}{8} =$
$408\frac{3}{8}$; $408\frac{3}{8} \times 2 = 816\frac{3}{4}$; $816\frac{3}{4} + 12 = 828\frac{3}{4}$; $828\frac{3}{4}$
$\times 3 \times 4\frac{1}{2} \times 2 = \$223.76\frac{1}{4}$, Ans.

25. $14\frac{7}{12} = \frac{175}{12}$; $5\frac{1}{2} = \frac{11}{2}$; $4\frac{1}{4} = \frac{17}{4}$; $2150\frac{2}{5} = \frac{10752}{5}$; $\frac{175}{12}$
$\times \frac{11}{2} \times \frac{17}{4} \times \frac{1728}{1} \times \frac{5}{10752} = \frac{1824984000}{6193152} = 294\frac{2429}{3584}$
bushels, Ans.

26. $10 \times 8 \times 6 = 480$; $8 \times 8 \times 8 = 512$; $512 - 480 = 32$
feet, Ans.

27. $7\frac{1}{2} \times 6 \times 5\frac{1}{2} = \frac{15}{2} \times \frac{6}{1} \times \frac{11}{2} = \frac{990}{4} = 247\frac{1}{2}$ cubic feet.
$9\frac{1}{2} \times 4\frac{1}{2} \times 5\frac{1}{2} = \frac{19}{2} \times \frac{9}{2} \times \frac{11}{2} = \frac{1881}{8} = 235\frac{1}{8}$; $247\frac{1}{2}$
$- 235\frac{1}{8} = 12\frac{3}{8}$; $1728 \times 12\frac{3}{8} = 21384$; $21384 \div 231$
$= 92\frac{4}{7}$ gallons, Ans.

The first cistern will contain $92\frac{4}{7}$ most gallons.

28. 31rd. $13\frac{3}{10}$ft., 41rd. $1\frac{9}{10}$ft., 38rd. $0\frac{1}{2}$ft., 45rd. $12\frac{7}{10}$ft. =
$524\frac{4}{5}$ft., $678\frac{2}{5}$ft., $627\frac{1}{5}$ft., $755\frac{1}{5}$ ft. =

2624	3392	3136	3776
5	5	5	5

The greatest common divisor of 2624, 3392, 3136, 3776, is $= 64$
The greatest common multiple of 5, 5, 5, 5, is $= 5$
Therefore $\frac{64}{5}$ will divide each of the fractions without a
remainder; thus $\frac{2624}{5} \div \frac{64}{5} = 41$; $\frac{3392}{5} \div \frac{64}{5} = 53$;
$\frac{3136}{5} \div \frac{64}{5} = 49$; $\frac{3776}{5} \div \frac{64}{5} = 59$. The number of
rails will therefore be $41 + 53 + 49 + 59 = 202$; 202
$\times 4 = 808$ rails.

$64 \div 5 = 12\frac{4}{5}$; $12\frac{4}{5} + \frac{7}{10} = 13\frac{1}{2}$ feet, length of the rails, Ans.

29. $\frac{5}{6}$, $\frac{3}{4}$, $\frac{9}{7}$ = $\frac{35}{56}$, $\frac{42}{56}$, $\frac{48}{56}$.

Least common multiple of the numerators = 1680

Greatest common divisor of the denominators = $\frac{1680}{56}$ = 30yds., Ans.

 We therefore find that 30 is the least whole number that can be divided by $\frac{5}{6}$, $\frac{3}{4}$, or $\frac{9}{7}$, without a remainder.

30. $100 \div 30 = 3\frac{1}{3}$ times.

31. 31A. 3R. 6p. = 5086p.; 39A. 2R. 37$\frac{1}{2}$p. = 6357$\frac{1}{2}$p.

$$\frac{5086 \times 2 = 10172}{6357\frac{1}{2} \times 2 = 12715} = \frac{4}{5}, \text{ Ans.}$$

32. $68 \times \$7\frac{11}{12} = \$538.33\frac{1}{3}$, Ans.

33. $8\frac{2}{3} \times \$42\frac{3}{5} = \$369.20$, Ans.

34. $\frac{3-3}{3-3} = 0$, Ans.

35. $20 \times 15 = 300$ft., contents of the upper part of the room.

 $20 + 15 = 35$; $35 \times 2 = 70$ft., length round the room.

 $70 \times 8\frac{1}{2} = 595$ feet, contents of the upright ceiling of the room.

 $2 \times 7 \times 3 = 42$ft., contents of the doors.

 $4 \times 5\frac{1}{2} \times 3\frac{1}{4} = 73\frac{1}{2}$ft., contents of the windows.

 $70 - 6 = 64$; $64 \times \frac{2}{3} = 42\frac{2}{3}$ft., contents of the mop-boards.

  NOTE. — We deduct 6 feet for the two doors.

 $595 + 300 = 895$; $42 + 73\frac{1}{2} + 42\frac{2}{3} = 158$.

 $895 - 158 = 737$ft.; $737 \div 9 = 81\frac{8}{9}$ square yards.

 $81\frac{8}{9} \times 6\frac{1}{4} = \frac{737}{9} \times \frac{25}{4} = \frac{18425}{36} = \$5.11\frac{29}{36}$, for plastering.

 $595 - 158 = 437$ft.; $437 \div 9 = 48\frac{5}{9}$ square yards.

 $48\frac{5}{9} \times .09 = \$4.37$, for papering.

 $48\frac{5}{9} \times 3 = 145\frac{2}{3} = \frac{437}{3}$; $2\frac{2}{3} = \frac{1.3}{5}$; $\frac{437}{3} \div \frac{1.3}{5} = \frac{437}{3} \times$
[$\frac{5}{13} = \frac{2185}{39} = \$2.80\frac{5}{39}$, for paper, Ans.

QUESTIONS TO BE PERFORMED BY ANALYSIS.

(Page 194.)

3. $30\frac{4}{15} = \frac{454}{15}$; $\frac{1728}{1} \times \frac{15}{454} = \frac{25920}{454}$; $7\frac{17}{30} = \frac{227}{30}$; $\frac{25920}{454}$
 $\times \frac{227}{30} = \frac{5883840}{13620} = \$432$, Ans.

4. $7\frac{17}{30} = \frac{227}{30}$; $\frac{432}{1} \times \frac{30}{227} = \frac{12960}{227}$; $\frac{1728}{1} \times \frac{227}{12960} = \frac{392256}{12960} = 30\frac{4}{15}$ tons, Ans.

5. $7\frac{17}{30} = \frac{227}{30}$; $\frac{432}{1} \times \frac{30}{227} = \frac{12960}{227}$; $30\frac{4}{15} = \frac{454}{15}$; $\frac{454}{15} \times \frac{12960}{227} = \frac{5883840}{3405} = \$1728$, Ans.

6. $30\frac{4}{15} = \frac{454}{15}$; $\frac{1728}{1} \times \frac{15}{454} = \frac{25920}{454}$; $\frac{432}{1} \times \frac{454}{25920} = 7\frac{17}{30}$ tons, Ans.

7. $7\frac{7}{10} = \frac{77}{10}$; $6\frac{4}{9} = \frac{58}{9}$; $\frac{58}{9} \times \frac{77}{10} = \frac{4466}{90} = 49\frac{28}{45}$ bushels, Ans.

8. $\frac{7}{11}$ of $\frac{17}{1} = \frac{119}{11} = 10\frac{9}{11}$; $10\frac{9}{11} + 15 = 25\frac{9}{11} = \frac{284}{11}$; $\frac{1728}{1} \times \frac{11}{284} = \frac{19008}{284} = \$66\frac{66}{71}$, each girl's share; $\frac{7}{11}$ of $66\frac{66}{71} = \frac{7}{11} \times \frac{19008}{284} = \frac{133056}{3124} = \$42\frac{42}{71}$, each boy's share.

9. $\frac{1449}{1} \times \frac{9}{7} = \frac{13041}{7} = 18.63$; $4\frac{3}{7} = \frac{31}{7}$; $\frac{1863}{1} \times \frac{31}{7} = \frac{57753}{7} = \$82.50\frac{3}{7}$, Ans.

10. $82.50\frac{3}{7} = \frac{57753}{7}$; $4\frac{3}{7} = \frac{31}{7}$; $\frac{57753}{7} \times \frac{7}{31} = \frac{404271}{217} = 18.63$; $\frac{1449}{1863} = \frac{7}{9}$, Ans.

11. $\frac{1449}{1} \times \frac{9}{7} = \frac{13041}{7} = 18.63$; $82.50\frac{3}{7} \div 18.63 = 4\frac{3}{7}$, Ans.

12. $82.50\frac{3}{7} = \frac{57753}{7}$; $4\frac{3}{7} = \frac{31}{7}$; $\frac{57753}{7} \times \frac{7}{31} = \frac{404271}{217} = 18.63$; $\frac{7}{9}$ of $\frac{1863}{1} = \frac{13041}{9} = \$14.49$, Ans.

13. $14\frac{7}{8} = \frac{119}{8}$; $\frac{500}{1} \times \frac{8}{119} = \frac{4000}{119}$; $9\frac{11}{12} = \frac{119}{12}$; $\frac{4000}{119} \times \frac{119}{12} = \frac{476000}{1428} = \$333.33\frac{1}{3}$, Ans.

14. $9\frac{11}{12} = \frac{119}{12}$; $333.33\frac{1}{3} = \frac{100000}{3}$; $\frac{100000}{3} \times \frac{12}{119} = \frac{1200000}{357}$; $14\frac{7}{8} = \frac{119}{8}$; $\frac{1200000}{357} \times \frac{119}{8} = \frac{142800000}{2856} = \$500$, [Ans.

15. $333.33\frac{1}{3} = \frac{100000}{3}$; $9\frac{11}{12} = \frac{119}{12}$; $\frac{100000}{3} \times \frac{12}{119} = \frac{1200000}{357}$; [$\frac{\cancel{50000}}{1} \times \frac{357}{\cancel{1200000}} = \frac{1785}{120} = 14\frac{7}{8}$ tons, Ans.

16. $14\frac{7}{8} = \frac{119}{8}$; $\frac{50000}{1} \times \frac{8}{119} = \frac{400000}{119}$; $333.33\frac{1}{3} = \frac{100000}{3}$; $\frac{400000}{119} \times \frac{119}{400000} = \frac{11900000}{1200000} = 9\frac{11}{12}$ tons, Ans.

17. $97\frac{7}{8} = \frac{783}{8}$; $\frac{3132}{1} \times \frac{8}{783} = \frac{25056}{783} = .32$; $763\frac{5}{8} \times .32 = \$244.36$, Ans.

18. $763\frac{5}{8} = \frac{6109}{8}$; $\frac{24436}{1} \times \frac{8}{6109} = \frac{195488}{6109} = .32$; $97\frac{7}{8} \times .32 = \$31.32$, Ans.

19. $763\frac{5}{8} = \frac{6109}{8}$; $24\frac{136}{1} \times \frac{8}{6109} = \frac{195488}{6109} = .32$; $31.32 \div .32 = 97\frac{7}{8}$gal., Ans.

20. $1975 \div 40 = 49\frac{3}{8}$; $49\frac{3}{8} \times 144 = 7110$lb., Ans.

21. $15.75 \div 17 = 92\frac{11}{17}$; $92\frac{11}{17} = \frac{1575}{17}$; $9\frac{1}{4} = \frac{37}{4}$; $\frac{1575}{17} \times \frac{37}{4} = \frac{58275}{68} = \$8.56\frac{47}{68}$, Ans.

22. $50\frac{4}{7} = \frac{354}{7}$; $\frac{1}{9} \times \frac{354}{7} = \frac{354}{63}$; $87\frac{3}{5} = \frac{438}{5}$; $\frac{354}{63} \times \frac{438}{5} = \frac{155052}{315} = \$492\frac{8}{35}$, Ans.

23. $78 \times 13 = 1014$; $13 + 7 = 20$; $1014 \div 20 = 50\frac{7}{10}$ days, Ans.

24. $10 \times 9 = 90$; $90 \div 15 = 6$ days, Ans.

25. $15 \times 6 = 90$; $90 \div 10 = 9$ days, Ans.

26. $10 \times 9 = 90$; $90 \div 6 = 15$ hours, Ans.

27. $17\frac{3}{11} = \frac{190}{11}$; $5\frac{3}{7} = \frac{38}{7}$; $\frac{38}{7} \times \frac{11}{190} = \frac{418}{1330}$; $97\frac{5}{9} = \frac{878}{9}$; $\frac{418}{1330} \times \frac{878}{9} = \frac{367004}{11970} = \$30\frac{208}{315}$, Ans.

28. $9\frac{3}{5} = \frac{48}{5}$; $19\frac{7}{9} = \frac{178}{9}$; $\frac{178}{9} \times \frac{48}{5} = \frac{8544}{45} = 189\frac{13}{15}$; $189\frac{13}{15} + 7 = 196\frac{13}{15}$; $9\frac{3}{5}$ tons $= 192$cwt.; $196\frac{13}{15} \div 192 = \$1\frac{73}{2880}$, Ans.

29. $9\frac{3}{5}$ tons $= 192$ cwt.; $192 \times 1\frac{3}{4} = 336$; $336 - 7 = \$329$, Ans.

30. $47\frac{3}{11} = \frac{520}{11}$; $2\frac{2}{3} = \frac{8}{3}$; $\frac{520}{11} \times \frac{8}{3} = \frac{4160}{33} = 126\frac{2}{33}$; $0.75 = \frac{3}{4}$; $126\frac{2}{33} \div \frac{3}{4} = 168\frac{8}{99}$ bushels, Ans.

31. $57\frac{9}{11} = \frac{636}{11}$; $\frac{636}{11} \times \frac{1}{15} = \frac{636}{165}$; $19\frac{7}{8} = \frac{159}{8}$; $\frac{636}{165} \times \frac{159}{8} = \frac{101124}{1320} = \$76\frac{67}{110}$, Ans.

32. $19\frac{7}{8} = \frac{159}{8}$; $76\frac{67}{110} = \frac{101124}{1320}$; $\frac{101124}{1320} \times \frac{8}{159} = \frac{808992}{209880} = \frac{636}{165}$; $57\frac{9}{11} = \frac{636}{11}$; $\frac{636}{11} \times \frac{165}{636} = 15$ cords, Ans.

33. $7\frac{3}{10} = \frac{73}{10}$; $47\frac{1}{3} = \frac{142}{3}$; $\frac{142}{3} \times \frac{73}{10} = \frac{10366}{30} = 345\frac{8}{15}$s. $= 17£.$ 5s. $6\frac{2}{3}$d., Ans.

34. $172£.$ 15s. $0\frac{2}{5}$d. $= \frac{207302}{5}$d.; $47\frac{1}{3} = \frac{142}{3}$; $\frac{207302}{5} \times \frac{3}{142} = \frac{621906}{710} = 875\frac{328}{355}$d. $= 3£.$ 12s. $11\frac{328}{355}$d., Ans.

35. $43\frac{6}{7} = \frac{307}{7}$; $\frac{307}{7} \times \frac{7}{13} = \frac{2149}{91} = \$23.61\frac{7}{13}$, Ans.

36. $17\frac{3}{8} = \frac{139}{8}$; $2\frac{7}{11} = \frac{29}{11}$; $\frac{29}{11} \times \frac{139}{8} = \frac{232}{1529}$; $\frac{232}{1529} \times \frac{50}{1} = \frac{11600}{1529} = \$7.58\frac{1018}{1529}$, Ans.

37. $87\frac{7}{4} = \frac{612}{7}$; $\frac{612}{7} \times \frac{7}{612} = 7$; $14\frac{7}{10} \times 7 = \$102.90$, Ans.

8*

38. $\frac{4375}{1} \times \frac{?}{?} = \frac{35000}{?} = 50$; $50 \times 10 = $ \$500, Ans.

39. $500 \div 10 = 50$; $\frac{4375}{8755} = \frac{7}{8}$ acre, Ans.

40. $71.87 \div 9 = 7.98\frac{5}{9} = \frac{7187}{9}$; $\frac{4}{7} \times \frac{7187}{9} = \frac{28748}{63} = $ \$4.56$\frac{20}{63}$, Ans.

41. $4.56\frac{20}{63} \times \frac{7}{4} = 7.98\frac{5}{9}$; $7.98\frac{5}{9} \times 9 = $ \$71.87, Ans.

42. $111 \times 19 = 2109$; $2109 \div 47 = 44\frac{41}{47}$ days, Ans.

43. $44\frac{41}{47} \times 47 = 2109$; $2109 \div 19 = 111$ days, Ans.

44. $\frac{2530}{1} \times \frac{1}{1} = \frac{10120}{4} = 25.30$; $25.30 \times 17 = $ \$430.10, Ans.

45. $430.10 \div 17 = 25.30$; $\frac{4}{11} \times \frac{2530}{1} = \frac{10120}{11} = $ \$9.20, Ans.

46. $\frac{2145}{1} \times \frac{16}{1} = \frac{11440}{7} = 16.34\frac{2}{7}$; $16.34\frac{2}{7} \times 7 = $ \$114.40, Ans.

47. $\frac{7}{16} \times \frac{11440}{7} = \frac{11440}{16} = $ \$7.15, Ans.

48. $19\frac{3}{7} = \frac{136}{7}$; $87\frac{3}{7} = \frac{612}{7}$; $\frac{136}{7} \times \frac{7}{612} = \frac{136}{612} = \frac{34}{153}$;

$$\left[\frac{34}{153} \times \frac{60}{1} \times \frac{7}{1} = \frac{14280}{153} = 93\frac{51}{153} \text{bu., Ans.} \right.$$

49. Smith will reap $\frac{1}{80}$ of the field in an hour; his wife will reap $\frac{1}{108}$ of the field in an hour. They will both reap $\frac{1}{80} + \frac{1}{108} = \frac{?}{?}$ of the field, in an hour. Then they will reap the whole field in $2\frac{160}{?}$ hours $= 45\frac{45}{47}$; $45\frac{45}{47} \div 8 = 5\frac{34}{47}$ days, Ans.

DECIMAL FRACTIONS.

(Art. 266, p. 199.)

1. Fifty-six thousandths.
2. One thousand three ten thousandths.
3. Two thousand seven hundred eighty-six ten thousandths.
4. Sixteen thousand three hundred two hundred thousandths.
5. Nine hundred seventy-five thousandths.
6. One and six hundred thirty one thousandths.
7. Forty-eight and seven hundredths.

8. One and three hundred fifteen thousandths.

9. Five and six thousand one ten thousandths.

10. Eighty-seven and six ten thousandths.

11. One and seven millionths.

12. Five and one hundred one thousand sixteen millionths.

13. One and three hundred twenty-seven millionths.

14. One millionth.

15. Sixteen and seven billionths.

16.	.13	21.	.001031	26.	465.14
17.	.006	22.	7.0017	27.	93.07
18.	.0019	23.	333.003	28.	24.000009
19.	.00406	24.	1.000001	29.	221.00009
20	.000001	25.	325.7	30.	49000.049

31.	79002000.105
32.	69015.00015
33.	80000.0083
34.	9000019019.19
35.	27.927
36.	49000000000000.000000000001
37.	21.0001
38.	87000.000087
39.	99099.000009009
40.	17.0117
41.	33.33
42.	47000.0000029
43.	15.04007
44.	11000.11
45.	17.000000000000081
46.	9.000000000057
47.	69000.349

ADDITION OF DECIMALS.
(ART. 268, p. 201.)

2.	7564.0052656	4.	7234.0968
3.	2071.449495	5.	6913.5477

(6.)	(7.)	(8.)
$15.06	$137.50	23000010.
107.09	55.63	1000.00005
1.625	1.375	27.000019
93.765	.875	7.5
Ans. $217.54	Ans. $195.38	Ans. 23001044.500069

(9.)	(10.)	(11.)
59.059	25.000007	3.75
25000.0025	145.643	11.7
5.000005	175.89	16.125
205.05	17.00348	Ans. 31.575
Ans. 25269.111505	Ans. 363.536487	

(12.)	(13.)
73.29	209000.000046
87.047	98207.0015
3005.0106	15.08
28.03	.0049
29000.005	Ans. 307222.086446
Ans. 32193.3826	

SUBTRACTION OF DECIMALS.

(Art. 270, p. 202.)

6.   9.49989 | 7.   88.9429 | 8.   .001

		(11.)
(9.)	(10.)	29004005.
97.7	315.0027	29000.
27.028	115.07	349200.00024
Ans. 70.672	Ans. 199.9327	378200.00024
		Ans. 28625804.99976

(12.)	(13.)	(14.)
1000000.	$19.	$400.
.000001	1.375	316.875
Ans. 999999.999999	Ans. $17.625	Ans. $83.125

(16.) 1000.

93.45

(15.)   124.

19000000.   244.285

.000000019   216.136

Ans. 18999999.999999981   577.871

Ans. 422.129

MULTIPLICATION OF DECIMALS.

(Art. 271, p. 203.) | 10. | 33.5175
3. | .438496 | 11. | 41448651.06
4. | .0949416 | 12. | .000019737
5. | .003721061 | 13. | $153.525
6. | 26137.65 | 14. | $18.4375
7. | 152.2756 | 15. | $46.95
8. | 43910.073 | 16. | $149.5125
9. | .00000081 | 17. | 3616.175

CONTRACTIONS IN MULTIPLICATION OF DECIMALS.

1. (Art. 272, p. 204.) 131634. | 3. | 1.
2. | 347890. | 4. | $7000.

(Art. 273, p. 206.)   (3.)

(2.) 325.701428   56.7534916

3938127   8296735

227990   28376746

6514   (4.) 843.7527   1702605

326   5714368   397274

261   6750022   34052

10   506252   5108

8   25313   114

Ans. 235.104   3375   45

84   Ans. 305.15944

59

4

Ans. 7285109.

DIVISION OF DECIMALS.

(Art. 274, p. 207.)		
3.	321.2	
4.	.758	
5.	.561	
6.	13.861	
7.	749.084	
8.	3124.3	
9.	.31243	
10.	312430000.	
11.	.000031243	
12.	312.43	
13.	31243000000.	
14.	.31243	
15.	.31243	
16.	32000.	
17.	.000032	
18.	.5403+	
19.	140yd.	
20.	$50.	
21.	64753000000.	

CONTRACTIONS IN DIVISION OF DECIMALS.

(Art. 275, p. 208.)	
1.	3.1675
2.	9.1605
3.	.000070461
4.	.00070461
5.	.0070461
6.	.070461
7.	.70460
8.	.0000070460.
9.	.0001965
10.	$3.50
11.	$0.1025

2. (Art. 276, p. 209.) 4.95445
3.   426.1043
4.   8.7938+
5.   9876.54321

REDUCTION OF DECIMALS.

(Art. 277, p. 210.)

2. $\frac{875}{1000} = \frac{7}{8}$

3. $\frac{9375}{10000} = \frac{15}{16}$

4. $\frac{8125}{100000} = \frac{13}{160}$

5. $\frac{75}{100000} = \frac{3}{4000}$

6. $31\frac{75}{100} = 31\frac{3}{4}$

7. $96\frac{24}{1000} = 96\frac{3}{125}$

8. $163\frac{4}{100} = 163\frac{1}{25}$

9. $1001\frac{4375}{10000} = 1001\frac{7}{16}$

10. $1457\frac{222}{1000} = 1457\frac{111}{500}$

11. $19678\frac{86}{100} = 19678\frac{9}{25}$

12. $9163\frac{8755}{10000} = 9163\frac{1751}{2000}$

2. (Art. 278, p. 211.) .625
3.   .5
4.   .09375
5.   .076932

6.	19.125	11.	$4.3125
7.	$315.875	12.	$60.1875
8.	1163.75	13.	.51
9.	.625	14.	2.9875
10.	.370625		

(ART. **279**, p. **213**.)

(2.)
20 | 9.00
.45, Ans.

(3.)
25 | 14.
4 | 3.56
20 | 15.89
.7945, Ans.

(4.)
16 | 12.
16 | 8.75
25 | 21.546875
4 | 2.861875
.71546875, Ans.

(5.)
4 | 3.00
4 | 1.7500
.4375, Ans.

(6.)
12 | 9.00
3 | 2.75
5½ | 2.91666666
40 | 35.53030303
8 | 5.88825757
.73603219+, Ans.

(7.)
2 | 1.
4 | 2.5
63 | 3.625
.0575396+, Ans.

(8.)
2 | 1.0
8 | 0.50
4 | 0.0625
0.015625, Ans.

(9.)
40 | 16.
4 | 2.4
.6, Ans.

(10.)
40 | 175
4.375, Ans.

(11.)
4 | 3.755
Ans. .98875

(12.)
10 | 6.0
60 | 34.6
60 | 25.576
Ans., .4262+

	(13.)		(14.)
25	20.0	2	1.5
4	2.8	4	2.75
	243.7, Ans.		156.6875, Ans.

(15.)

4	3.	4	1.0	4	2.
12	11.75	12	9.25	12	5.5
20	19.97916+	20	16.77083+	20	17.4583+
	.9989583+, Ans.		.8385416+, Ans.		.8729166+, [Ans.

$$.9989583+$$
$$.8385416+$$
$$.8729166+$$
$$\overline{2.710416+}$$

(Art. 280, p. 214.)

2. $.625 \times 12 = 7.5$d.; $.5 \times 4 = 2.0$; $7\frac{1}{2}$d., Ans.

3. $.6725 \times 4 = 2.69$; $.69 \times 25 = 17.25$; $25 \times 16 = 4$; 2qr. 17lb. 4oz., Ans.

4. $.9375 \times 4 = 3.75$; $.75 \times 4 = 3$; 3qr. 3na., Ans.

5. $.7895 \times 8 = 6.316$; $.316 \times 40 = 12.64$; $.64 \times 16\frac{1}{2} = 10.56$; $.56 \times 12 = 6.72$; $72 = \frac{18}{25}$; 6fur. 12rd. 10ft. $6\frac{18}{25}$in., Ans.

6. $.9378 \times 4 = 3.7512$; $.7512 \times 40 = 30.048$; $.048 \times 272\frac{1}{4} = 13.068$; $.068 \times 144 = 9.792$; $.792 = \frac{99}{125}$; 3R. 30p. 13ft. $9\frac{99}{125}$in., Ans.

7. $.5615 \times 63 = 35.3745$; $.3745 \times 4 = 1.498$; $.498 \times 2 = .996$; $.996 \times 4 = 3.984$; $.984 = \frac{123}{125}$; 35gal. 1qt. 0pt. $3\frac{123}{125}$gi., Ans.

8. $.367 \times 365\frac{1}{4} = 134.046\frac{3}{4}$; $.046\frac{3}{4} \times 24 = 1.122$; $.122 \times 60 = 7.32$; $.32 \times 60 = 19.2$; $.2 = \frac{1}{5}$; 134da. 1h. 7m. $19\frac{1}{5}$sec., Ans.

9. .6923828125 × 4 = 2.76953125; .76953125 × 25 = 19.-
   23828125; .23828125 × 16 = 3.8125; .8125 × 16 =
   13; 2qr. 19lb. 3oz. 13dr., Ans.

10. .015625 × 4 = .0625; .0625 × 8 = .5; .5 × 2 = 1;
    1p., Ans.

11. .55 × 5 = 2.75; .75 × 4 = 3; 2qr. 3na., Ans.

12. .6 × 4 = 2.4; .4 × 40 = 16; 2R. 16p., Ans.

MISCELLANEOUS EXERCISES.

<table>
<tr><td colspan="2">(1.)</td><td colspan="2">(2.)</td></tr>
<tr><td>25</td><td>18.</td><td>25</td><td>14.</td></tr>
<tr><td>4</td><td>2.72</td><td>4</td><td>3.56</td></tr>
<tr><td></td><td>7.68</td><td></td><td>19.89</td></tr>
<tr><td></td><td>$ 11.75</td><td></td><td>$ 9.25</td></tr>
<tr><td></td><td>3840</td><td></td><td>9945</td></tr>
<tr><td></td><td>5376</td><td></td><td>3978</td></tr>
<tr><td></td><td>768</td><td></td><td>17901</td></tr>
<tr><td></td><td>768</td><td></td><td>$ 183.9825 Ans.</td></tr>
<tr><td></td><td>$ 90.24, Ans.</td><td></td><td></td></tr>
</table>

<table>
<tr><td colspan="2">(3.)</td><td colspan="2">(4.)</td></tr>
<tr><td>40</td><td>15.000</td><td>40</td><td>15.000</td></tr>
<tr><td>4</td><td>2.37500</td><td>8</td><td>3.375000</td></tr>
<tr><td></td><td>39.59375</td><td></td><td>87.421875</td></tr>
<tr><td></td><td>87.375</td><td></td><td>578.75</td></tr>
<tr><td></td><td>19796875</td><td></td><td>437109375</td></tr>
<tr><td></td><td>27715625</td><td></td><td>611953125</td></tr>
<tr><td></td><td>11878125</td><td></td><td>699375000</td></tr>
<tr><td></td><td>27715625</td><td></td><td>611953125</td></tr>
<tr><td></td><td>31675000</td><td></td><td>437109375</td></tr>
<tr><td></td><td>3459.50390625 =</td><td></td><td>50595.41015625 =</td></tr>
<tr><td></td><td>$ 3459.503$\frac{39}{72}$ Ans.</td><td></td><td>$ 50595.41$\frac{1}{64}$ Ans.</td></tr>
</table>

9

(5.)

12	9.00	12	3.50
	18.75		2.29⅙

2.29⅙

16875
3750
3750

3125

42.96875
.053

12890625
21484375

2.27734375 = $2.277¹¹⁄₃₂, Ans.

(6.)

12	7.168
	1.597¼

6.5

7985
9582
21⅜

10.3826⅜
12

4.5920

10ft. 4.592in., Ans.

(7.)

29½ = 29.5
4.316)29.500(6h. 50m. 6+sec.,
25896   [Ans.

3604
60

4.316)216240(50m.
21580

440
60

4.316)26400(6+sec.
25896

(8.)

4	2.0
4	1.500

5.375
5.625

26875
10750
32250
26875

30.234375 =
$30.234⅜, Ans.

(9.)

25	7.00
4	3.28

4.82

17

81.94

5.875

40970

57358

65552

40970

$481.39750 =

$ 481.39¾, Ans.

(10.)

40	25.000
4	3.625

176.90625

75.375

88453125

123834375

53071875

88453125

123834375

13334.30859375 =

$13334.308¹⁹⁄₃₂, Ans.

(11.)

17.625

12.75

88125

123375

35250

17625

224.71875 =

$ 224.718¾, Ans.

(12.)

19ft. 3in. = 19.25

15ft. 9in. = 15.75

9625

13475

9625

1925

303.1875

144

7500

7500

1875

27.0000

303ft. 27in., Ans.

13. 14ft. 6in. = 14.5; 12ft. 6in. = 12.5; 8ft. 9in. = 8.75;
    14.5 + 12.5 = 27; 27 × 2 = 54; 8.75 × 54 = 472.5;
    472.5 ÷ 9 = 52½ yards, Ans.

14. 10ft. 7in. = 127in.; 5ft. 10in. = 70in.; 127 × 70 × 4 = 35560; 35560 ÷ 144 = 246$\frac{7}{18}$ft., Ans.

<table>
<tr><td colspan="2" align="center">(15.)</td><td align="center">(16.)</td></tr>
<tr><td>4 | 3.00</td><td>4 | 2.0</td><td>2 | 1.0</td></tr>
<tr><td>4 | 2.75</td><td>112.5</td><td>4 | 2.500</td></tr>
<tr><td>4.6875</td><td></td><td>1.625</td></tr>
</table>

112.5 ÷ 4.6875 = 24, Ans.   1.82 ÷ 1.625 = 1.12

$1.12 × 63 = $70.56, Ans.

17. $15.06 ÷ 125.5 = $.12, Ans.

18. $\dfrac{4\ |\ 3}{17.75}$ × 35.75 = 634.562\frac{1}{2}$, Ans.

19. $87.25 × 7$\frac{47}{63}$ = 675.84\frac{8}{63}$, Ans.

20. 34ft. 9in. = 34.75; 1ft. 3in. = 1.25; 1ft. 6in. = 1.5; 34.75 × 1.25 × 1.5 = 65.15625ft., Ans.

<table>
<tr><td align="center">(21.)</td><td align="center">(22.)</td></tr>
<tr><td>4 | 1.00</td><td>$477.72 ÷ 9 = $53.08, Ans.</td></tr>
<tr><td>| 18.25</td><td></td></tr>
</table>

36.50 ÷ 18.25 = $2.00, Ans.

<table>
<tr><td align="center">(23.)</td><td align="center">(24.)</td></tr>
<tr><td>5.375</td><td>2$\frac{3}{8}$ = 2.375; 3$\frac{7}{8}$ = 3.875; 1$\frac{1}{16}$ = 1.0625;</td></tr>
<tr><td>1.78</td><td>2.375 + 3.875 + 1.0625 = 7.3125</td></tr>
<tr><td>1.1875</td><td align="right">17.625</td></tr>
<tr><td>1.125</td><td align="right">365625</td></tr>
<tr><td>1.275</td><td align="right">146250</td></tr>
<tr><td>2.625</td><td align="right">438750</td></tr>
<tr><td>13.3675 =</td><td align="right">511875</td></tr>
<tr><td>13.367\frac{1}{2}$, Ans.</td><td align="right">73125</td></tr>
<tr><td></td><td align="right">128.8828125 =</td></tr>
<tr><td></td><td>128.882\frac{13}{16}$, Ans.</td></tr>
</table>

(25.)

$15.25 \times 8.4 \times 10 = 1281;$

$1281 \times 1728 = 2213568;$

$2213568 \div 231 = 9682\frac{6}{11}$ gal.

$9682\frac{6}{11} \div 63 = 152$ hhd. $6\frac{6}{11}$ gal., Ans.

(26.)

$\begin{array}{c|c} 25 & 7.00 \\ 4 & 2.28 \\ \hline \end{array}$

3.57

13.625

1785
714
2142
1071
357

$\overline{\$ 48.64125} =$

$\$ 48.641\frac{1}{4}$, Ans.

(27.)

$\begin{array}{c|c} 40 & 35.000 \\ 4 & 3.87500 \\ \hline \end{array}$

37.96875

125.75

18984375
26578125
18984375
7593750
3796875

$\overline{4774.5703125} =$

$\$ 4774.570\frac{5}{16}$, Ans.

(28.)

$\begin{array}{c|c} 25 & 21.00 \\ 4 & 2.84 \\ \hline \end{array}$

17.71

11.25

8855
3542
1771
1771

$\$ \overline{199.23\frac{3}{4}}$, Ans.

(30.) 97.625

$7\frac{48}{63}$

683375

75930$\frac{35}{63}$

$759.305\frac{35}{63} =$

$\$ 759.305\frac{35}{63}$, Ans.

(29.)

9.375

3.37

65625
28125
28125

31.59375

7.75

15796875
22115625
22115625

$\overline{244.8315625} =$

$\$ 244.851\frac{9}{16}$, Ans.

9*

(31.)	(32.)	(33.)
4|3.00	25| 3.50	40|15.000
4|3.7500	4| 1.14	8| 3.375000
7.9375	20|15.285	17.421875
4.75	27.76425	1725.875
396875	183.62	87109375
555625	5552850	121953125
317500	16658550	139375000
37.703125 =	8329275	87109375
\$37.703⅛, Ans.	22211400	34843750
	2776425	121953125
	5098.0715850 =	17421875
	\$5098.071¹¹⁷⁄₂₀₀, Ans.	30067.978515625 =
		\$30067.978³³⁄₆₄, Ans.

(34.)	(35.)
40|15.	25|12.5
4| 3.375	4| 2.5
A. 17.84375	20|15.625
\$624.53125 ÷ 17.84375 =	19.78125
\$35, Ans.	\$494.53125 ÷ 19.78125 =
	\$24.999¹¹²⁄₃₂₉₇, Ans.

(36.)   40)1004.75
A. 25.11875
4
R. .47500
40
p. 19.00000
Ans. 25A. 0R. 19p.

37. 20.5 × 12.75 × 7.6 = 1986.45 cubic feet;
    1986.45 ÷ 128 = 15 cords 66²⁄₂₀ feet.

38. 31 × .08¼ = \$2.55¾; \$2.75 × 7½ = \$20.62½; \$20.62½
    + \$2.55¾ = \$23.18¼; 23.18¼ ÷ .62½ = 37²³⁄₂₅₀yd.,
    [Ans.

39. 3.50 ÷ 40 = 8¾ bushels.

$\frac{9}{20}$, $\frac{3}{10}$ = $\frac{9}{20}$, $\frac{6}{20}$; $\frac{9}{20}$ + $\frac{6}{20}$ = $\frac{15}{20}$; $\frac{28}{20}$ − $\frac{15}{20}$ = $\frac{13}{20}$bu.; 8¾ ÷ 5 = 1¾ bushels.

1¾bu. × 6 = 10½bu.; 1¾bu. × 9 = 15¾bu.

Ans. 8¾ bushels of oats; 10½ bushels corn; 15¾ bushels wheat.

Proof: $ 0.75 ×10½ = $ 7.87,5; $ 2 × 15¾ = $ 31.50; $ 3.50 + $ 7.87,5 + $ 31.50 = $ 42.87,5.

40. 250.35 ÷ 2 = 125.175; 125.175 ÷ 8 = 15.646875; 15.646875 + 1 = 16.646875 times, Ans.

41. ¼ + ⅙ = $\frac{7}{12}$; 1⅔ − $\frac{7}{12}$ = $\frac{5}{12}$; $\frac{5}{12}$ × $\frac{1}{5}$ = $\frac{1}{12}$, part for Mary.

$\frac{7}{12}$ + $\frac{1}{12}$ = $\frac{8}{12}$; 1⅔ − $\frac{8}{12}$ = $\frac{4}{12}$ = ⅓, part for wife.

¼ − $\frac{1}{12}$ = ⅙ = $ 2243.26, part Mary received less than James.

Therefore $ 2243.26 × 6 = $ 13459.56, amount.

$ 13459.56 × ¼ = $ 3364.89, James' part.
$ 13459.56 × ⅓ = $ 4486.52, William's part.
$ 13459.56 × $\frac{1}{12}$ = $ 1121.63, Mary's part.
$ 13459.56 × ⅓ = $ 4486.52, wife's part.

INFINITE OR CIRCULATING DECIMALS.
(ART. 296, p. 220.)

3. .$\dot{6}$ = $\frac{6}{9}$ = ⅔, Ans.

4. 1.6$\dot{2}$ = 1.62$\dot{1}$ = 1$\frac{621}{999}$ = 1$\frac{23}{37}$, Ans.

5. .5$\dot{3}$ = $\frac{5\frac{3}{9}}{10}$ = $\frac{48}{90}$ = $\frac{8}{15}$, Ans.

6. .$\dot{7}$6923$\dot{0}$ = $\frac{769230}{999999}$ = $\frac{10}{13}$, Ans.

7. .59$\dot{2}\dot{5}$ = $\frac{5\frac{925}{999}}{10}$ × $\frac{999}{999}$ = $\frac{592}{999}$ = $\frac{16}{27}$, Ans.

8. 31.6$\dot{2}$ = 31.62$\dot{1}$ = 31$\frac{621}{999}$ = 31$\frac{23}{37}$, Ans.

9. .008$\dot{4}$9713$\dot{3}$ = $\frac{8\frac{497133}{999999}}{1000}$ = $\frac{8497125}{999999000}$ = $\frac{23}{2706}$, Ans.

(ART. **297**, p. **222**.)

3. $\frac{1}{11} = .\dot{0}\dot{9}$, Ans.
4. $\frac{38}{1320} = \frac{13}{440} = 13 \div 440 = .0295\dot{4}$, Ans.
5. $\frac{107}{253} = 107 \div 253 = .\dot{4}22924901185770750988\dot{1}$, Ans.
6. $13\frac{17}{45}$; $17 \div 45 = .3\dot{7}$; $13.3\dot{7}$, Ans.
7. $\frac{166}{19536} = \frac{83}{9768}$; $83 \div 9768 = .008497133\dot{3}$, Ans.
8. $2 \div 29 = .\dot{0}682758620689655172413793103\dot{4}\dot{8}$, Ans.

TRANSFORMATION OF REPETENDS.

(ART. **302**, p. **223**.)

(2.)		(3.)	
$8.6\dot{7}\dot{1}$	$= 8.671671167\dot{1}$	$1.5\dot{2}$	$= 1.52525\dot{2}$
$1.007\dot{1}$	$= 1.00710071007\dot{1}$	$8.715\dot{6}$	$= 8.71567\dot{1}$
$8.5\dot{2}$	$= 8.52525252525\dot{2}$	$3.56\dot{7}$	$= 3.56777\dot{7}$
$7.61632\dot{5}$	$= 7.616325616325$	$1.37\dot{8}$	$= 1.37878\dot{7}$

(4.)		(5.)	
$.000\dot{7}$	$= .00070707\dot{0}$	$.3\dot{1}2\dot{3}$	$= .312312\dot{3}$
$.14\dot{1}41\dot{4}$	$= .14141441\dot{4}$	$3.\dot{2}\dot{7}$	$= 3.272727\dot{2}$
$887.\dot{1}$	$= 887.111111111\dot{1}$	$5.0\dot{2}$	$= 5.022222\dot{2}$

(6.)	
$17.088\dot{4}$	$= 17.0884848484848484848484848484\dot{4}$
$1563.092\dot{9}$	$= 1563.092992992992992992992992\dot{9}$
$15.\dot{1}234\dot{5}$	$= 15.1234512345123451234512345123451\dot{2}$

ADDITION OF CIRCULATING DECIMALS.

(ART. **303**, p. **224**.)

(2.)	(3.)		
$3.555555\dot{5}$	$\frac{1}{3} = .\dot{3}$	$= .33333\dot{3}$	
$7.651651\dot{6}$	$\frac{1}{7} = .\dot{1}4285\dot{7}$	$= .\dot{1}4285\dot{7}$	
$1.765765\dot{7}$	$\frac{1}{9} = .\dot{1}$	$= .11111\dot{1}$	
$6.173737\dot{3}$		Ans. $.58730\dot{1}$	
$51.777777\dot{7}$			
$3.700000\dot{0}$			
$27.631631\dot{6}$			
$1.003003\dot{0}$			
Ans. 103.2591227			

(4.)

27.5̇6̇ = 27.5675675675675
5.6̇32̇ = 5.6326326326326
6.7̇ = 6.7777777777777
16.3̇56̇ = 16.3565656565656
.7̇1̇ = .7111111111111
6.1̇234̇ = 6.1234123412341

Ans. 63.1690670868888

(5.)

.165002 = .165002
31.64 = 31.64
1.6̇ = 1.6666666
.34634̇ = .3463444
13. = 13.

Ans. 46.8180131

(6.)

.8̇7̇ = .878787
.8̇ = .888888
.876̇ = .876876

Ans. 2.644555

(7.)

.3̇ = .3333333333
.45 = .45
.4̇5̇ = .4545454545
.3̇51̇ = .3513513513
.6468 = .6468
.6468̇ = .6468888888
.64̇68̇ = .6468686868
.6̇468̇ = .6468468468

Ans. 4.1766345618

(8.)

1.25 = 1.250000000
3.4̇ = 3.444444444
.6̇37̇ = .637373737
7.88̇5̇ = 7.885555555
7.875 = 7.875000000
7.8̇75̇ = 7.875875875
11.1̇ = 11.111111111

Ans. 40.079360722

(9.)

131.613 = 131.613
15.001̇ = 15.0011
67.134̇ = 67.1344
1000.6̇3̇ = 1000.6333

Ans. 1214.3818

(10.)

$5.\dot{1}634\dot{5} = 5.\dot{1}6345163451634516345$
$8.\dot{6}38\dot{1} = 8.\dot{6}381638163816381638\dot{1}$
$8.7\dot{5} = 3.7575757575757575757\dot{5}$

Ans. $17.\dot{5}591912084737409030\dot{2}$

SUBTRACTION OF CIRCULATING DECIMALS.

(ART. **304**, p. **225**.)

(2.)	(3.)
$7.\dot{1} = 7.1\dot{1}$	$31\dot{5}.8\dot{7} = 315.\dot{8}7587587587\dot{5}$
$5.0\dot{2} = 5.02$	$78.\dot{0}37\dot{8} = 78.\dot{0}378037800378$
Ans. $\overline{2.0\dot{8}}$	Ans. $\overline{237.\dot{8}38072095497}$

(4.)

$\frac{2}{9} = .\dot{2} = .\dot{2}22222\dot{2}$
$\frac{1}{7} = .\dot{1}4285\dot{7} = .\dot{1}42857$

Ans. $.\dot{0}79365$

(5.)

$16.13\dot{4}\dot{7} = 16.13\dot{4}\dot{7}$
$11.08\dot{8}\dot{4} = 11.08\dot{8}\dot{4}$

Ans. $5.04\dot{6}\dot{2}$

(6.)

$18.16\dot{7}\dot{8} = 18.16\dot{7}\dot{8}$
$3.\dot{2}\dot{7} = 3.2\dot{7}2\dot{7}$

Ans. $14.895\dot{1}$

(7.)

$8.\dot{1}2\dot{3} = 3.\dot{1}2312\dot{3}$
$.\dot{7}\dot{1} = .\dot{7}1717\dot{1}$

Ans. $2.\dot{4}05951$

(8.)

$\frac{3}{7} = .\dot{4}2857\dot{1} = .\dot{4}2857\dot{1}$
$\frac{2}{11} = .\dot{1}\dot{8} = .\dot{1}81818$

Ans. $.\dot{2}4675\dot{3}$

(9.)

$\frac{4}{9} = .\dot{4} = .44444\dot{4}$
$\frac{2}{7} = .\dot{2}8571\dot{4} = .\dot{2}8571\dot{4}$

Ans. $.\dot{1}5873\dot{0}$

(10.)

$\frac{9}{17} = .\dot{5}294117647058823\dot{5}$
$\frac{6}{17} = .\dot{3}529411764705882\dot{3}$

Ans. $.\dot{1}764705882352941\dot{1}$

(11.)

5.1̇2345̇ = 5.12345123451234512345123451234512345̇1̇
2.3̇523456̇ = 2.3̇52345652345652345652345652345̇6̇

Ans. 2.7̇7110558216669277779888885999994̇

MULTIPLICATION OF CIRCULATING DECIMALS.

(Art. **305**, p. **226**.)

(2.)

87.3258̇6̇
437
———————
61128106
26197759₇
34930346₃₄
———————
Ans. 381.6̇140338̇

3. 582.3̇47̇ = 582³⁴⁷⁄₉₉₉ = ⁵⁸¹⁷⁶⁵⁄₉₉₉ ; .0̇3̇ = ³⁄₉₀ = ₃₀¹⁄ ; ⁵⁸¹⁷⁶⁵⁄₉₉₉
× ₃₀¹⁄ = ⁵⁸¹⁷⁶⁵⁄₅₉₉₇₀ = ¹¹⁶³⁵³⁄₅₉₉₄ ; ¹¹⁶³⁵³⁄₅₉₉₄ = 19.4115782449̇,
Ans.

4. 3.14̇5̇ = 3.1⁴⁵⁄₉₉ = 3.1₁₁⁵⁄ = ³⁴⁶⁄₁₁₀ ; 4.2̇97̇ = 4²⁹⁷⁄₉₉₉ = 4₃₇¹¹⁄ =
¹⁵⁹⁄₃₇ ; ³⁴⁶⁄₁₁₀ × ¹⁵⁹⁄₃₇ = ⁵⁵⁰¹⁴⁄₄₀₇₀ = 13.51̇69533̇, Ans.

(5.)

.2̇85714̇
28
———————
2285714
571428₅
———————
Ans. 8.0̇00000̇

(6.)

.461607̇142857̇
20
———————
9.232142̇857142̇
4
———————
0.928571̇428571̇
25
———————
4642857̇142855̇
18571428571428
———————
23.214285̇714283̇
Ans. 9cwt. 0qr. 23+lb.

(7.) .2̇84931506̇
365
———————
1424657534
1709589041₀
854794520₅₄
———————
Ans. 104.0̇00000000̇

DIVISION OF CIRCULATING DECIMALS.

(Art. 306, p. 226.)

2. $345.\dot{8} = 345\frac{8}{9}$; $.\dot{6} = \frac{6}{9} = \frac{2}{3}$; $345\frac{8}{9} \div \frac{2}{3} = 518\frac{5}{6} = 518.$
$.8\dot{3}$, Ans.

3. $234.\dot{6} = 234\frac{6}{9}$; $.\dot{7} = \frac{7}{9}$; $234\frac{2}{3} \div \frac{7}{9} = 301\frac{5}{7} = 301.7142$-
$8\dot{5}$, Ans.

4. $13.51695\dot{3} = 13\frac{5}{10} + \frac{169533}{99999990} = \frac{1351683880}{99999990}$; $3.1\dot{4}\dot{5}$
$= 3\frac{1}{10} + \frac{45}{990} = \frac{3114}{9900}$; $\frac{1351683880}{99999990} \div \frac{3114}{9900} =$
$4.\dot{2}9\dot{7}$, Ans.

5. $381.614033\dot{8} = 381\frac{6140}{10000} + \frac{338}{9990000} = \frac{3812324198000}{9990000000}$;
$4.3\dot{7} = 4\frac{37}{100}$; $\frac{3812324198000}{9990000000} \div 4\frac{37}{100} = 87.3258\dot{6}$,
Ans.

6. $.42857\dot{1} = \frac{428571}{999999}$; $.625 = \frac{5}{8}$; $\frac{428571}{999999} \div \frac{5}{8} = .685714\dot{2}$,
Ans.

7. $2.\dot{3}7\dot{0} = 2\frac{370}{999} = \frac{2368}{999}$; $4.\dot{9}2307\dot{6} = 4\frac{923076}{999999} = \frac{4923072}{999999}$;
$\frac{2368}{999} \div \frac{4923072}{999999} = .48\dot{1}$, Ans.

8. $\frac{9}{99} \div \frac{230769}{999999} = .\dot{3}\dot{9}$, Ans.

9. $316.31015 \div \frac{1}{3} = 948.93045$, Ans.

10. $100006 \div \frac{2}{3} = 150009$, Ans.

11. $.\dot{3}\dot{6} = \frac{36}{99} = \frac{4}{11}$; $.2\dot{5} = \frac{2}{10} + \frac{5}{90} = \frac{23}{90}$; $\frac{4}{11} \div \frac{23}{90} =$
$1.\dot{4}229249011857707509881\dot{1}$, Ans.

CONTINUED FRACTIONS.

(Art. 309, p. 229.)

(3.) $\frac{261}{82} = 82)261(3$
      $\underline{246}$
      $15)82(5$
        $\underline{75}$
         $7)15(2$
          $\underline{14}$
          $1)7(7$
          $\underline{7}$

First approx. val. $= \frac{3}{1} = 3$.

$\frac{3 \times 5 + 1}{1 \times 5 + 0} = \frac{16}{5} = 3\frac{1}{5}$.

$\frac{16 \times 2 + 3}{5 \times 2 + 1} = \frac{35}{11} = 3\frac{2}{11}$.

$\frac{35 \times 7 + 16}{11 \times 7 + 5} = \frac{261}{82} = 3\frac{15}{82}$.

Ans. $3, 3\frac{1}{5}, 3\frac{2}{11}, 3\frac{15}{82}$, real val.

(4.) $\frac{29}{77}$ = 29)77(2   First approx. val. = $\frac{1}{2}$.

       58          $\frac{1 \times 1 + 0}{2 \times 1 + 1} = \frac{1}{3}$'

   19)29(1

      19          $\frac{1 \times 1 + 1}{3 \times 1 + 2} = \frac{2}{5}$'

     10)19(1

      10          $\frac{2 \times 1 + 1}{5 \times 1 + 3} = \frac{3}{8}$'

      9)10(1

       9          $\frac{3 \times 9 + 2}{8 \times 9 + 5} = \frac{29}{77}$, original val.

      1)9(9

       9   Ans. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{29}{77}$.

(5.) $\frac{1327}{1631}$ = 1327)1631(1

           1327

         304)1327(4

First approx. val. = $\frac{1}{1}$.   1216

$\frac{1 \times 4 + 0}{1 \times 4 + 1} = \frac{4}{5}$·   111)304(2

               222

$\frac{4 \times 2 + 1}{5 \times 2 + 1} = \frac{9}{11}$·   82)111(1

              82

$\frac{9 \times 1 + 4}{11 \times 1 + 5} = \frac{13}{16}$·   29)82(2

              58

$\frac{13 \times 2 + 9}{16 \times 2 + 11} = \frac{35}{43}$·   24)29(1

              24

$\frac{35 \times 1 + 13}{43 \times 1 + 16} = \frac{48}{59}$·   5)24(4

              20

$\frac{48 \times 4 + 35}{59 \times 4 + 43} = \frac{131}{161}$·   4)5(1

              4

$\frac{131 \times 1 + 48}{161 \times 1 + 59} = \frac{179}{220}$·   1)4(4

              4

$\frac{179 \times 4 + 131}{220 \times 4 + 161} = \frac{847}{1041}$· Ans. $\frac{1}{1}$, $\frac{4}{5}$, $\frac{9}{11}$, $\frac{13}{16}$, $\frac{35}{43}$, $\frac{48}{59}$,

                            [$\frac{131}{161}$, $\frac{179}{220}$, $\frac{847}{1041}$.

10

(6.)

$\frac{829}{347} = 347)829(2$

$\frac{2}{1} = 2$, first approx. val.

$\frac{694}{}$

$\overline{135)347(2}$

$2 \times 2 + 1 = 5$
$\overline{1 \times 2 + 0} = \overline{2} = 2\frac{1}{2}$.

$\frac{270}{}$

$\overline{77)135(1}$

$5 \times 1 + 2 = 7$
$\overline{2 \times 1 + 1} = \overline{3} = 2\frac{1}{3}$.

$\frac{77}{}$

$\overline{58)77(1}$

$7 \times 1 + 5 = 12$
$\overline{3 \times 1 + 2} = \overline{5} = 2\frac{2}{5}$.

$\frac{58}{}$

$\overline{19)58(3}$

$12 \times 3 + 7 = 43$
$\overline{5 \times 3 + 3} = \overline{18} = 2\frac{7}{18}$.

$\frac{57}{}$

$\overline{1)19(19}$

$43 \times 19 + 12 = 819$
$\overline{18 \times 19 + 5} = \overline{347} =$

$\frac{19}{}$

$[2\frac{125}{347},$

Ans. $2, 2\frac{1}{2}, 2\frac{1}{3}, 2\frac{2}{5}, 2\frac{7}{18}, 2\frac{125}{347}$.

(7.) $\frac{1}{13\frac{568}{1000}} = \frac{1000}{13568}$.

$\frac{1}{13}$ = first approx. val.

$1000)13568(13$

$\frac{13000}{}$

$1 \times 1 + 0 = 1$
$\overline{13 \times 1 + 1} = \overline{14}$.

$\overline{568)1000(1}$

$1 \times 1 + 1 = 2$
$\overline{14 \times 1 + 13} = \overline{27}$.

$\frac{568}{}$

$\overline{432)568(1}$

$2 \times 3 + 1 = 7$
$\overline{27 \times 3 + 14} = \overline{95}$.

$\frac{432}{}$

$\overline{136)432(3}$

$7 \times 5 + 2 = 37$
$\overline{95 \times 5 + 27} = \overline{502}$.

$\frac{408}{}$

$\overline{24)136(5}$

$37 \times 1 + 7 = 44$
$\overline{502 \times 1 + 95} = \overline{597}$.

$\frac{120}{}$

$\overline{16)24(1}$

$\frac{16}{}$

$44 \times 2 + 37 = 125$
$\overline{579 \times 2 + 44} = \overline{1396}$.

$\overline{8)16(2}$

$\frac{16}{}$

Ans. $\frac{1}{13}, \frac{1}{14}, \frac{2}{27}, \frac{7}{95}, \frac{37}{502}, \frac{44}{597}, \frac{125}{1396}$.

(8.) $1.27 = \frac{127}{100}$.   First approx. val. $= \frac{1}{1}$.

$100)127(1$
  $\underline{100}$
  $27)100(3$
   $\underline{81}$
   $19)27(1$
    $\underline{19}$
    $8)19(2$
     $\underline{16}$
     $3)8(2$
      $\underline{6}$
      $2)3(1$
       $\underline{2}$
       $1)2(2$
        2

$\begin{aligned}&1 \times 3 + 1 = 4\\&1 \times 3 + 0 = \overline{3}\end{aligned} = 1\frac{1}{3}.$

$\begin{aligned}&4 \times 1 + 1 = 5\\&3 \times 1 + 1 = \overline{4}\end{aligned} = 1\frac{1}{4}.$

$\begin{aligned}&5 \times 2 + 4 = 14\\&4 \times 2 + 3 = \overline{11}\end{aligned} = 1\frac{3}{11}.$

$\begin{aligned}&14 \times 2 + 5 = 33\\&11 \times 2 + 4 = \overline{26}\end{aligned} = 1\frac{7}{26}.$

$\begin{aligned}&33 \times 1 + 14 = 47\\&26 \times 1 + 11 = \overline{37}\end{aligned} = 1\frac{10}{37}.$

$\begin{aligned}&47 \times 2 + 33 = 127\\&37 \times 2 + 26 = \overline{100}\end{aligned} = 1\frac{27}{100}.$

Ans. $\frac{1}{1}, \frac{4}{3}, \frac{5}{4}, \frac{14}{11}, \frac{33}{26}, \frac{47}{37}, \frac{127}{100} = 1, 1\frac{1}{3}, 1\frac{1}{4}, 1\frac{3}{11}, 1\frac{7}{26}, 1\frac{10}{37}, 1\frac{27}{100}.$

RATIO.

REDUCTION AND COMPARISON OF RATIOS.

(Art. 328, p. 232.)

2. $63 : 72 = \frac{63}{72} = \frac{7}{8}$, Ans.

3. $66 : 24 = \frac{66}{24} = \frac{11}{4}$, Ans.

4. $4 \times 6 \times 3 : 8 \times 9 \times 2 = 72 : 144 = \frac{72}{144} = \frac{1}{2}$, Ans.

5. $19 \times 5 \times 2 \times 3 : 15 \times 12 \times 38 = 570 : 684 = \frac{570}{684}$
   $= \frac{95}{114}$, Ans.

(Art. 329, p. 233.)

3. $\frac{5}{6} : \frac{4}{7} = \frac{5}{6} \times \frac{7}{4} = \frac{35}{24} = 35 : 24$, Ans.

4. $13\frac{1}{2} : 27 = \frac{13\frac{1}{2}}{27} = \frac{27}{54} = \frac{1}{2} = 1 : 2$, Ans.

5. $6.25 : 3.125 = \frac{6.25}{3.125} = \frac{2}{1} = 2 : 1$, Ans.

6. $\left.\begin{array}{r}4 : 16 \\ 25 : 10\end{array}\right\} = 4 \times 25 : 16 \times 10 = 100 : 160 = \frac{100}{160} =$

$[\frac{5}{8} = 5 : 8$, Ans.

7. $\left.\begin{array}{r}3 : 6 \\ 9 : 27 \\ 108 : 12\end{array}\right\} = 3 \times 9 \times 108 = 2916 : 6 \times 27 \times 12 =$

$[1924 = \frac{2916}{1924} = \frac{3}{2} = 3 : 2$, Ans.

8. $\left.\begin{array}{r}12\frac{1}{2} : 6\frac{1}{4} \\ 76.5 : 25.5\end{array}\right\} = 12.5 \times 76.5 = 956.25 : 6.25 \times 25.5 =$

$[159.375 = \frac{956.25}{159.375} = \frac{6}{1} = 6 : 1$, Ans.

(Art. **330**, p. **233**.)

3. $39 : 13 = \frac{39}{13} = 3$, Ans.

4. $2\frac{1}{2} : 9 = \frac{2\frac{1}{2}}{9} = \frac{5}{18}$, Ans.

5. $21 : 21 = \frac{21}{21} = 1$, Ans.

6. $\frac{1}{3} \times \frac{1}{6} \times \frac{200}{1} = \frac{200}{18} = 1\frac{9}{9} : \frac{1}{2} \times \frac{50}{1} = \frac{50}{2} = 25 ; 1\frac{9}{9}$
   $\times \frac{1}{25} = \frac{4}{9}$, Ans.

7. $24 : 6 = \frac{6}{24} = \frac{1}{4}$, Ans.

8. $4 : 36 = \frac{4}{36} = \frac{1}{9}$, Ans.

9. 94A. 2R. 16p. $= 15136$p.; 11A. 3R. $= 1880$p.; $1880 :$
   $15136 = \frac{1880}{15136} = \frac{235}{1892}$, Ans.

10. $17 : 9 = \frac{17}{9} = 1\frac{8}{9}$; $39 : 19 = \frac{39}{19} = 2\frac{1}{19}$; therefore the
    ratio of 39 to 19 is the larger.

11. $36 \times 4 \times 3 = 432 : 12 \times 16 \times 2 = 384 = \frac{432}{384} = \frac{9}{8}$;
    $60 \div (3 \times 5) = 4$; $(20 \times 2) \div 8 = 5$; $\frac{9}{8} - \frac{4}{5} =$
    $= \frac{13}{40}$, Ans.

12. $.0\dot{2} = \frac{2}{90}$; $2.50\dot{3} = 2\frac{503}{999} = \frac{2501}{999} = \frac{90}{2} \times \frac{2501}{999} = \frac{225090}{1998}$
    $= 112\frac{73}{111}$, Ans.

13. $\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$; $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$; $\frac{10}{1} \times \frac{1}{12} = \frac{10}{12} = \frac{5}{6}$.
    $4 : 5 = \frac{4}{5}$; $\frac{5}{6} - \frac{4}{5} = \frac{1}{30}$, Ans. That is, the first ratio is
    the greater by $\frac{1}{30}$.

14. $220 : 500 = \frac{220}{500} = \frac{11}{25}$, Ans.

15. $36 : 180 = \frac{1}{5}$, Ans.

16.) 4900)11283(2

$\qquad$ 9800

$\qquad$ 1483)4900(3

First approx. ratio $= \frac{1}{2}$.   4449

$1 \times 3 + 0 = 3$ 451)1483(3

$\overline{2} \times 3 + 1 = \overline{7}$ 1353

$3 \times 3 + 1 = 10$ 130)451(3

$\overline{7} \times 3 + 2 = \overline{23}$ 390

$10 \times 3 + 3 = 33$ 61)130(2

$\overline{23} \times 3 + 7 = \overline{76}$ 122

$33 \times 2 + 10 = 76$ 8)61(7

$\overline{76} \times 2 + 23 = \overline{175}$ 56

$76 \times 7 + 33 = 565$ 5)8(1

$\overline{175} \times 7 + 76 = \overline{1301}$ 5

$565 \times 1 + 76 = 641$ 3)5(1

$\overline{1301} \times 1 + 135 = \overline{1436}$ 3

$641 \times 1 + 555 = 1196$ 2)3(1

$\overline{1436} \times 1 + 1301 = \overline{2737}$ 2

$1196 \times 1 + 641 = 1837$ 1)2(2

$\overline{2737} \times 1 + 1436 = \overline{4173}$ 2

$1837 \times 2 + 1196 = 4870$

$\overline{4173} \times 2 + 2637 = \overline{11083}$

Ans. $\frac{1}{2}$, $\frac{3}{7}$, $\frac{10}{23}$, $\frac{33}{76}$, $\frac{76}{175}$, $\frac{565}{1301}$, $\frac{641}{1436}$, $\frac{1196}{2737}$, $\frac{1837}{4173}$, $\frac{4870}{11083}$.

10*

(17.) 1000000)3141592(3
$$3000000$$
$$\overline{141592)1000000(7}$$
$$991144$$
$$\overline{8856)141592(15}$$

First approx. ratio $= \frac{3}{1}$.

$3 \times 7 + 1 = 22$
$\overline{1} \times 7 + 0 = \overline{7}$

$22 \times 15 + 3 = 333$
$\overline{7} \times 15 + 1 = \overline{106}$

$333 \times 1 + 22 = 355$
$\overline{106} \times 1 + 7 = \overline{113}$

$355 \times 84 + 333 = 30153$
$\overline{113} \times 84 + 106 = \overline{9492}$

$30153 \times 6 + 355 = 181273$
$\overline{9492} \times 6 + 113 = \overline{57065}$

$181273 \times 2 + 30153 = 392699$
$\overline{57065} \times 2 + 9492 = \overline{123622}$

$$\begin{array}{l}
8856 \\
\overline{53032} \\
44280 \\
\overline{8752)8856(1} \\
8752 \\
\overline{104)8752(84} \\
832 \\
\overline{432} \\
416 \\
\overline{16)104(6} \\
96 \\
\overline{8)16(2} \\
16
\end{array}$$

Ans. $\frac{3}{1}$, $\frac{22}{7}$, $\frac{333}{106}$, $\frac{355}{113}$, $\frac{30153}{9492}$, $\frac{181273}{57065}$, $\frac{392699}{123622}$ =

3, $3\frac{1}{7}$, $3\frac{15}{106}$, $3\frac{16}{113}$, $3\frac{1677}{9492}$, $3\frac{19978}{57065}$, $3\frac{21833}{123622}$.

ANALYSIS BY RATIO.

(ART. 331, p. 235.)

(2.) $80

$$\underline{17}$$

     $1360   = 17A.

(2R. : 1A. $= \frac{1}{2}$); $\frac{1}{2}$ of 80 =   40   = 2R.
(1R. : 2R. $= \frac{1}{2}$); $\frac{1}{2}$ of 40 =   20   = 1R.
(20r. : 1R. $= \frac{1}{2}$); $\frac{1}{2}$ of 20 =   10   = 20r.
(10r. : 20r. $= \frac{1}{2}$); $\frac{1}{2}$ of 10 =   5   = 10r.
(5r. : 10r. $= \frac{1}{2}$); $\frac{1}{2}$ of   5 =   2.50 = 5r.

     $1437.50, Ans.

(3.)
$$
\begin{array}{r}
\$2.50 \\
16 \\
\hline
\$40.00 = 16\text{cwt.}
\end{array}
$$

(2qr. : 1cwt. = $\frac{1}{2}$); $\frac{1}{2}$ of $2.50 = 1.25 = 2qr.
(1qr. : 2qr. = $\frac{1}{2}$); $\frac{1}{2}$ of $1.25 = .625 = 1qr.
(10lb. : 1cwt. = $\frac{1}{10}$); $\frac{1}{10}$ of $2.50 = .25 = 10lb.
$$\$42.125, \text{ Ans.}$$

(4.)
$$
\begin{array}{r}
\$14.00 \\
27 \\
\hline
\$378.00 = 27\text{cwt.}
\end{array}
$$

(1qr. : 1cwt. = $\frac{1}{4}$); $\frac{1}{4}$ of $14 = 3.50 = 1qr.
(20lb. : 1cwt. = $\frac{1}{5}$); $\frac{1}{5}$ of $14 = 2.80 = 20lb.
$$\$384.30, \text{ Ans.}$$

(5.)
$$
\begin{array}{r}
\$5.60 \\
7 \\
\hline
\$39.20 = 7\text{yd.}
\end{array}
$$

(2qr. : 1yd. = $\frac{1}{2}$); $\frac{1}{2}$ of $5.60 = 2.80 = 2qr.
(1qr. : 2qr. = $\frac{1}{2}$); $\frac{1}{2}$ of $2.80 = 1.40 = 1qr.
(2na. : 1qr. = $\frac{1}{2}$); $\frac{1}{2}$ of $1.40 = .70 = 2na.
$$\$44.10, \text{ Ans.}$$

(6.)
$$
\begin{array}{r}
\$20 \\
7 \\
\hline
\$140.00 = 7\text{ton.}
\end{array}
$$

(10cwt. : 1 ton = $\frac{1}{2}$); $\frac{1}{2}$ of $20 = 10.00 = 10cwt.
(2cwt. : 10cwt. = $\frac{1}{5}$); $\frac{1}{5}$ of $10 = 2.00 = 2cwt.
(1cwt. : 2cwt. = $\frac{1}{2}$); $\frac{1}{2}$ of $2 = 1.00 = 1cwt.
(2qr. : 1cwt. = $\frac{1}{2}$); $\frac{1}{2}$ of $1 = .50 = 2qr.
$$\$153.50, \text{ Ans.}$$

(8.) $4.40
                      19
                 ———————
                 $83.60 = 19yd.
(2qr. : 1yd. = ½); ½ of $4.40 = 2.20 =   2qr.
(1qr. : 1yd. = ¼); ¼ of $4.40 = 1.10 =   1qr.
(2na. : 1qr. = ½); ½ of $1.10 = .55 =   1na.
                 ———————
                 $87.45, Ans.

(9.) $32
                      24
                 ———————
                 $768
(3R. : 1A. = ¾); ¾ of $32 =   24
(20p. : 1A. = ⅛); ⅛ of $32 =   4
                 ———————
                 $796, Ans.

(10.)   200
                    $0.30
                 ———————
                 $60.00 = 200lb.
(4oz. : 1lb. = ¼) = ¼ of $0.30 = .075 =   4oz.
                 ———————
                 $59.925, Ans.

(11.)   714
                     15s.
                 ———————
                 3570
                 714
                 ———————
                 10710s.
(6d. : 1s. = ½); ½ of 714 =   357
                 20)11067
                 ———————
                 553£. 7s., Ans.

(12.) $450
$$\frac{2}{\$900} = 2y.$$
(6mo. : 1y. = ½); ½ of $450   = 225   = 6mo.
(3mo. : 6mo. = ½); ½ of $225   = 112.50 = 3mo.
(15da. : 3mo. = ⅙); ⅙ of $112.50 = 18.75 = 15da.
$$\overline{\$1256.25}, \text{ Ans.}$$

(13.)
$$\$80.50 \times 25 = \$2012.50 \quad = 25A.$$
(2R. : 1A. = ½); ½ of $80.50   =   40.25   = 2R.
(20r. : 2R. = ¼); ¼ of $40.25   =   10.06+ = 20r.
(10r. : 20r. = ½); ½ of $10.06+ =   5.03+ = 10r.
(5r. : 10r. = ½); ½ of $ 5.03+ =   2.51+ = 5r.
$$\overline{\$2070.35+}, \text{ Ans.}$$

(14.)   498
(2s. : 1£. = ¹⁄₁₀); ¹⁄₁₀ of 498   = 49£. 16s.
(6d. : 2s. = ¼); ¼ of 49£. 16s. = 12£. 9s.
$$\overline{62£. \quad 5s.}, \text{ Ans.}$$

(16.)
35gal. 2qt. 1pt. : 11gal. 3qt. 1pt. = 3 ;
$5.83¾ × 3 = $17.51¼, Ans.

(17.)
12yd. 1qr. 2na. : 24yd. 3qr. = ½ ;
$49.50 × ½ = $24.75, Ans.

(18.)
73bu. 3pk. : 14bu. 3pk. = 5 ;
17bu. 2pk. 4qt. × 5 = 88bu. 0pk. 4qt., Ans.

(19.)
$9.75 : $3.25 = ⅓ ;
1T. 2cwt. 2qr. 15lb. × ⅓ = 7cwt. 2qr. 5lb., Ans.

(20.)
4h. : 1h. 20m. = 3 ;
27m. 3fur. 20rd. × 3 = 82m. 2fur. 20rd., Ans.

SIMPLE PROPORTION.

(Art. **338**, p. **240**.)

3. 16A. : 197A. : : $720 : $8865, Ans.

$$\frac{197 \times \overset{48}{\cancel{720}}}{\cancel{16}} = \$8865, \text{ Ans.}$$

4. $8865 : $720 : : 137A. : 16A., Ans.

$$\frac{720 \times 137}{8865} = 16\text{A., Ans.}$$

5. 15hhd. : 84hhd. : : $17595 : $985.32, Ans.

$$\frac{84 \times 17595}{15} = \$985.32, \text{ Ans.}$$

6. $12 : $40 : : $6 : $12.

$$\frac{\overset{20}{\cancel{40}} \times \cancel{6}}{\underset{2}{\cancel{12}}} = \$20, \text{ Ans.}$$

7. 15 men : 10 men : : 45 days : 30 days, Ans.

$$\frac{10 \times \overset{3}{\cancel{45}}}{\underset{1}{\cancel{15}}} = 30 \text{ days, Ans.}$$

8. 7 + 9 = 16 : 8 + 4 = 12 : : 12 : 9.

$$\frac{12 \times 12}{16} = 9, \text{ Ans.}$$

9. 3 men : 9 men : : 17 days : 51 days, Ans.

10. 17 days : 51 days : : 3 men : 9 men; 9 − 3 = 6 men, Ans.

11. 5½ rods : 160 rods : : 1 rod : $29\frac{1}{11}$ rods, Ans.

12. $100 : $850 : : $6 : $51, Ans.

13. $6 : $32 : : $100 : $533.33⅓, Ans.

14. 20gal. : 180gal. : : 167lb. : 1503lb., Ans.

15. 2ft. : 3ft. : : 75ft. : 112½ft., Ans.

16. $4.75 : $160 : : 36 miles : $1212\frac{12}{19}$ miles, Ans.

17. 8 days : 12 days : : 100 men : 150 men, Ans.

18. $\frac{7}{12}$yd. : $\frac{1}{2}$yd. : : $\$\frac{7}{20}$: $\$0.48$, Ans.

19. 36A. 3R. : 21A. 3R. 20p. : : $\$1260$: $\$750$, Ans.

20. 10pwt. 18gr. : 2000lb. : : $\$10$: $\$446511\frac{27}{43}$, Ans.

21. $4\frac{1}{2}$yd. : $13\frac{1}{2}$yd. : : $\$9.75$: $\$29.25$, Ans.

22. $2\frac{1}{4}$in. : 144in. : : 1in. : $57\frac{3}{5}$in., Ans.

23. $\frac{7}{16} : \frac{9}{32} :$: 51£. : 10£. 18s. $6\frac{2}{9}$d., Ans.

24. 13° 10′ 35″ : 360° : : 1 day : 27da. 7h. 43m.+, Ans.

25. 7lb. : 12lb. : : $\$\frac{3}{4}$: $\$1.28\frac{4}{7}$, Ans.

26. $\$1.75$: $\$213.50$: : 7lb. : 8cwt. 2qr. 4lb., Ans.

27. 7oz. : 7lb. 11oz. : : 30£. : 407£. 2s. $10\frac{2}{3}$d., Ans.

28. $\$600$: $\$500$: : 6 months : 5 months, Ans.

29. $\$7.50$: $\$8.00$: : 7oz. : $7\frac{7}{15}$oz., Ans.

30. 1 man : 1000 men : : $3\frac{3}{4} \times 1\frac{7}{8} = \frac{225}{32}$yd. : $7031\frac{1}{4}$yd.

    $1\frac{1}{4}$yd. : 1yd. : : $7031\frac{1}{4}$yd. : 5625yd., Ans.

31. 10h. : 14h. : : 9 days : $12\frac{3}{5}$ days, Ans.

32. $75 - 40 = 35$gal. : 500gal. : : 1h. : 14h. 17m. $8\frac{4}{7}$sec., Ans.

33. $\$0.56$: $\$120.96$: : 1gal. : 216gal.

    $216 \div 12 = 18$doz., Ans.

34. 20m. : 1m. : : 1cist. : $\frac{1}{20}$cist.

    40m. : 1m. : : 1cist. : $\frac{1}{40}$cist.

    75m. : 1m. : : 1cist. : $\frac{1}{75}$.

    $\frac{1}{20} + \frac{1}{40} + \frac{1}{75} = \frac{53}{600}$.

    $\frac{53}{600}$cist. : 1cist. : : 1m. : 11m. $19\frac{13}{53}$sec., Ans.

35. 5da. : 1da. : : 1 field : $\frac{1}{5}$ field; 6da. : 1da. : : 1 field : $\frac{1}{6}$ field.

    $\frac{1}{5} + \frac{1}{6} = \frac{11}{30}$ field; $\frac{11}{30}$ field : 1 field : : 1da. : $2\frac{8}{11}$da., Ans.

36. 8ft. : 24ft. : : 6 men : 18 men.

    Then 6 days : 12 days : : 18 men : 36 men, Ans.

37. A can do $\frac{1}{20}$ of the labor in 1 day, A and C can do $\frac{1}{12}$ of it; therefore, C alone can do $\frac{1}{12} - \frac{1}{20} = \frac{1}{30}$ of it.

    Then $\frac{1}{30}$ work : 1 work : : 1da. : 30da., Ans.

38. 700 men : 1 man : : 184000lb. : $262\frac{6}{7}$lb.

    Then 5lb. : $262\frac{6}{7}$lb. : : 1 week : 52 weeks 4 days, Ans.

39. $\frac{3}{4} \times 3 \times 25 = 56\frac{1}{4}$lb.; $56\frac{1}{4}$lb. : 3150lb. : : 1 week : 56 weeks, Ans.

40. $8 \times 8 = 64$in. : $20 \times 16 \times 144 = 46080$in.
64in. : 46080in. : : 1 tile : 720 tiles, Ans.

41. $10 \times 9 \times 4 = 360$ cubic inches in each stone.
$80 \times 20 \times 2\frac{1}{2} \times 1728 = 6220800$ cubic inches in the wall.
360in. : 6220800in. : : 1 stone : 17280 stones, Ans.

42. 1T. 7cwt. 3qr. 20lb. $= 2795$lb.; 13T. 5cwt. 2qr. $= 26550$lb.
2795lb. : 26550lb. : : $9.50 : $90.24+, Ans.

43. 61.3lb. : 1lb. : : $44.99,42 : $0.734, Ans.

44. 1hhd. : .15hhd. : : $2.39 : $0.3585, Ans.

45. .75 ton : 1 ton : : $15 : $20, Ans.

46. .5yd. : 6yd. : : 10yd. : 120yd., Ans.

47. 10h. : 12h. : : 15da. : 18da., Ans.

48. 9 months : 5 months : : 450 men : 250 men.
$450 - 250 = 200$ men, Ans.

49. As the hour and minute hand pass each other 11 times in 12 hours, and as they are together at 12 o'clock, it is evident they will next pass each other in $\frac{1}{11}$ of 12 hours $= 1$h. 5m. $27\frac{3}{11}$sec., Ans.
11h. : 12h. : : 1h. : 1h. 5m. $27\frac{3}{11}$sec., Ans.

BY ANALYSIS.

50. If A and B can perform a piece of labor in $5\frac{5}{11}$ days, it is evident that in 1 day they would do $\dfrac{1}{5\frac{5}{11}} = \frac{11}{60}$ of the work. If B and C can do the work in $6\frac{2}{3}$ days, in 1 day they would perform $\dfrac{1}{6\frac{1}{2}} = \frac{3}{20}$ of the work. If A and C can do the work in 6 days, in 1 day they would perform $\frac{1}{6}$ of it. It then appears that A, B, and C, by laboring each 2 days, will perform $\frac{11}{60} + \frac{3}{20} + \frac{1}{6} = \frac{30}{60} = \frac{1}{2}$ of the work; and therefore, by laboring 1 day each, they would do $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$ of it. And if $\frac{1}{4}$ of the labor be performed in 1 day, it is evident that the whole work will be performed in 4 days, Ans.

If A, B, and C, can do $\frac{1}{4}$ of the work in one day, and A and B can do $\frac{11}{60}$, it is evident that C can do $\frac{1}{4} - \frac{11}{60} =$

$\frac{4}{60} = \frac{1}{15}$ of it in 1 day, and therefore will be 15 days in performing the whole. B and C can do $\frac{3}{20}$ of it in a day; therefore A can do $\frac{1}{4} - \frac{3}{20} = \frac{1}{10}$ of it in a day, or he will be 10 days in doing the whole. A and C can do $\frac{1}{6}$ of the work in a day; therefore B can do $\frac{1}{4} - \frac{1}{6} = \frac{1}{12}$ in a day, or, in performing the whole labor, he will be 12 days, Ans.

FORM OF STATEMENT.

$5\frac{5}{11}$ days : 1 day : : 1 work : $\frac{11}{60}$ work $=$ A and B.

$6\frac{2}{3}$ days : 1 day : : 1 work : $\frac{3}{20}$ work $=$ B and C.

6 days   : 1 day : : 1 work : $\frac{1}{6}$ work $=$ A and C.

$\qquad\qquad\qquad\qquad\quad \frac{1}{2}$ work $= \overline{2\,A,\,2\,B,\,2\,C} =$

$\qquad\qquad\qquad\qquad\quad \frac{1}{4}$ work $=$ A, B, and C.

$\frac{1}{4} - \frac{11}{60} = \frac{1}{15}$ work : 1 work : : 1 day : 15 days $=$ C.

$\frac{1}{4} - \frac{3}{20} = \frac{1}{10}$ work : 1 work : : 1 day : 10 days $=$ A.

$\frac{1}{4} - \frac{1}{6} = \frac{1}{12}$ work : 1 work : : 1 day : 12 days $=$ B.

$\qquad\quad \frac{1}{4}$ work : 1 work : : 1 day : 4 days $=$ A, B, and C.

(ART. 339, p. 243.)

(2.)

246A. 1R. 32p. $=$ 39432p.; $3 + 4 + 5 = 12$.

12 : 3 : : 39432p. : 61A. 2R. 18p. $\Big)$

12 : 4 : : 39432p. : 82A. 0R. 24p. $\Big\}$ Ans.

12 : 5 : : 39432p. : 102A. 2R. 30p. $\Big)$

(3.)

$4\frac{1}{4} = 4\frac{5}{20} = \frac{85}{20};$

$6\frac{1}{2} = 6\frac{10}{20} = \frac{130}{20};$

$6\frac{3}{5} = 6\frac{12}{20} = \frac{132}{20};$

$7\phantom{\frac{1}{2}} = 7\phantom{\frac{1}{2}} = \frac{140}{20};$

$\overline{24\frac{7}{20}} = \frac{487}{20}.$

$\frac{487}{20} : \frac{85}{20} : : 319 : 55\frac{335}{487}\Big)$

$\frac{487}{20} : \frac{130}{20} : : 319 : 85\frac{75}{487}\Big|$

$\frac{487}{20} : \frac{132}{20} : : 319 : 86\frac{228}{487}\Big\}$ Ans.

$\frac{487}{20} : \frac{140}{20} : : 319 : 91\frac{243}{487}\Big)$

11

4. Gold 9 + silver $\frac{1}{2}$ + copper $\frac{1}{2}$ = 10.

$\left.\begin{array}{l} 10 : 9 : : 1\text{oz. }1\text{pwt. }12\text{gr.} : 19\text{pwt. }8\frac{2}{5}\text{gr.} = \text{gold.} \\ 10 : \frac{1}{2} : : 1\text{oz. }1\text{pwt. }12\text{gr.} : 1\text{pwt. }1\frac{1}{5}\text{gr.} = \text{silver.} \\ 10 : \frac{1}{2} : : 1\text{oz. }1\text{pwt. }12\text{gr.} : 1\text{pwt. }1\frac{1}{5}\text{gr.} = \text{copper.} \end{array}\right\}$ Ans.

5. Silver 9 + copper 1 = 10 ; 192gr. × 20 = 3840gr.

$$10 : 9 : : 3840\text{gr.} : 7\text{oz. }4\text{pwt. silver.}$$
$$10 : 1 : : 3840\text{gr.} : 16\text{pwt. copper.}$$

6. First, 1 ; second, $1\frac{1}{3} \times 1 = 1\frac{1}{3}$; third, $1\frac{2}{3} \times 1\frac{1}{3} = 2\frac{2}{9}$;
$1 = \frac{9}{9}$; $1\frac{1}{3} = \frac{12}{9}$; $2\frac{2}{9} = \frac{20}{9}$; $\frac{9}{9} + \frac{12}{9} + \frac{20}{9} = \frac{41}{9}$.

$$41 : 9 : : \$600 : \$131\frac{49}{41} = \text{first man receives.}$$
$$41 : 12 : : \$600 : \$175\frac{25}{41} = \text{second man receives.}$$
$$41 : 20 : : \$600 : \$292\frac{28}{41} = \text{third man receives.}$$
Proof, $\$131\frac{49}{41} + \$175\frac{25}{41} + \$292\frac{28}{41} = \$600.$

7. 98 + 86 + 64 = 248.

$\left.\begin{array}{l} 248 \text{ tons} : 98 \text{ tons} : : 93 \text{ tons} : 36\frac{3}{4} \text{ A's tons.} \\ 248 \text{ tons} : 86 \text{ tons} : : 93 \text{ tons} : 32\frac{1}{4} \text{ B's tons.} \\ 248 \text{ tons} : 64 \text{ tons} : : 93 \text{ tons} : 24 \text{ C's tons.} \end{array}\right\}$ Ans.

8. 15 + 32 = 47 miles ; 25 + 32 = 57 miles.

$\left.\begin{array}{l} 47\text{mi.} : 15\text{mi.} : : 160\text{mi.} : 51\frac{3}{47}\text{mi. A.} \\ 57\text{mi.} : 25\text{mi.} : : 160\text{mi.} : 70\frac{10}{57}\text{mi. B.} \end{array}\right\}$ Ans.

COMPOUND PROPORTION.

(ART. 340, p. 245.)

(2.)

$\left.\begin{array}{l} 80 \text{ days} : 20 \text{ days} \\ 9 \text{ hours} : 12 \text{ hours} \end{array}\right\} : : 117 \text{ miles} : 104 \text{ miles, Ans.}$

$$\frac{\overset{2}{\cancel{20}} \times \overset{4}{\cancel{12}} \times 117}{\underset{3}{\cancel{9}} \times \underset{8}{\cancel{30}}} = 104 \text{ miles, Ans.}$$

(3.)

$$\left.\begin{array}{l} \text{6 men} \ : \ \text{24 men} \\ \text{9 hours:} \ \ \text{8 hours} \\ \text{20 feet} \ : \text{200 feet} \\ \text{6 feet} \ : \ \ \text{8 feet} \\ \text{4 feet} \ : \ \ \text{6 feet} \end{array}\right\} :: 16 \text{ days} : 1137\tfrac{1}{9} \text{ days, Ans.}$$

OPERATION.

$$\frac{24 \times 8 \times 200 \times 8 \times 6 \times 16}{6 \times 9 \times 20 \times 6 \times 4} = \frac{29491200}{25920} = 1137\tfrac{1}{9} \text{ days, Ans.}$$

(4.)

$$\left.\begin{array}{l} \$100 \quad : \$500 \\ 12 \text{ months} : 4 \text{ months} \end{array}\right\} :: \$6 : \$10, \text{ Ans.}$$

(5.)

$$\left.\begin{array}{l} \$6 \quad : \$10 \\ 4 \text{ months} : 12 \text{ months} \end{array}\right\} :: \$100 : \$500, \text{ Ans.}$$

(6.)

$$\left.\begin{array}{l} \$500 : \$100 \\ \$6 \quad : \$10 \end{array}\right\} :: 12 \text{ months} : 4 \text{ months, Ans.}$$

(7.)

$$\left.\begin{array}{l} \$500 \quad : \ \$100 \\ 4 \text{ months} : 12 \text{ months} \end{array}\right\} :: \$10 : \$6, \text{ Ans.}$$

(8.)

$$\left.\begin{array}{l} \text{9 comp.} : \ \text{5 comp.} \\ \text{10 hours} : \text{11 hours} \\ \text{25 sheets} : \text{36 sheets} \\ \text{24 pages} : \text{16 pages} \\ \text{44 lines} \ : \text{50 lines} \\ \text{40 letters} : \text{45 letters} \end{array}\right\} :: 16 \text{ days} : 12 \text{ days, Ans.}$$

(9.)

$$\left.\begin{array}{l} \text{60 men} \ : \ \text{12 men} \\ \text{30 feet} \ : \text{300 feet} \\ \text{6 feet} \ : \ \ \text{8 feet} \\ \text{3 feet} \ : \ \ \text{6 feet} \\ \text{8 hours:} \ \text{12 hours} \end{array}\right\} :: 15 \text{ days} : 120 \text{ days, Ans.}$$

CANCELLING.

$$\frac{12 \times \overset{10}{300} \times \cancel{8} \times \cancel{6} \times \cancel{12} \times \overset{1}{\cancel{15}}}{\underset{4}{\cancel{60}} \times \underset{1}{\cancel{30}} \times \cancel{6} \times \underset{1}{\cancel{3}} \times \cancel{8}} = 120 \text{ days, Ans.}$$

(10.)

16 horses : 32 horses } : : 84 bushels : 336 bushels, Ans.
24 days : 48 days

CANCELLING.

$$\frac{\overset{2}{\cancel{32}} \times \overset{2}{\cancel{48}} \times 84}{\underset{1}{\cancel{16}} \times \underset{1}{\cancel{24}}} = 336 \text{ bushels, Ans.}$$

(11.)

575 pounds : 775 pounds } : : $24.58 : $14.135 + Ans.
150 miles : 64 miles

(12.)

7¼oz. = 7.25 ; 4s. 2d. = 50d. ; 5s. 6d. = 66d. ; 1s. 2d. = 14d.

66d. : 50d. } : : 7.25oz. : 16¹¹⁸/₆₂₇oz., Ans.
4¾d. : 14d.

CANCELLING.

$$\frac{\overset{2}{\cancel{50}} \times \overset{7}{\cancel{14}} \times 7.25}{\underset{33}{\cancel{66}} \times \underset{19}{\cancel{4.75}}} = 16\tfrac{118}{627}\text{oz., Ans.}$$

(13.)

24 men : 496 men
9 hours : 11 hours
7 hard. : 4 hard. } : : 5½ days : 132 days, Ans.
465 feet : 337½ feet
3⅖ feet : 5⅗ feet
2⅓ feet : 3½ feet

CONJOINED PROPORTION.

(ART. 341, p. 247.)

(2.)

FORM OF STATEMENT.

100A. Bradford = 120A. Haverhill;
 50A. Haverhill = 65A. Methuen;
150A. Methuen.

OPERATION.

$$\frac{100 \times 50 \times 150 = 750000}{120 \times 65 = \overline{7800}} = 96\tfrac{2}{13}A., \text{ Ans.}$$

(3.)

FORM OF STATEMENT.

10lb. cheese = 7lb. butter;
11lb. butter = 2bu. corn;
11bu. corn = 8bu. rye;
 4bu. rye   = 1 cord wood;
10 cords wood.

OPERATION.

$$\frac{10 \times 11 \times 11 \times 4 \times 10 = 48400}{7 \times 2 \times 8 \times 1 = \overline{112}} = 432\tfrac{1}{7}lb., \text{ Ans.}$$

(4.)

STATEMENT.

12 men = 25 women.
 5 women = 6 boys.
75 boys.

OPERATION.

$$\frac{12 \times 5 \times 75}{25 \times 6} = 30 \text{ men, Ans.}$$

11*

(5.)

STATEMENT.

$$6 \text{ gallons } = \ 5 \text{ imperial gal.}$$
$$10 \text{ imp. gal.} = \ 6 \text{ Velts.}$$
$$26 \text{ Velts } = 16 \text{ Vedros.}$$
$$63 \text{ gallons.}$$

OPERATION.

$$\frac{5 \times 6 \times 16 \times 63}{6 \times 10 \times 26} = \frac{30240}{1560} = 19\frac{5}{13} \text{ Vedros, Ans.}$$

(6.)

STATEMENT.

$$7 \text{ Boston } = \ 8 \text{ Buffalo.}$$
$$10 \text{ Buffalo } = 14 \text{ Chicago.}$$
$$21 \text{ Chicago } = 25 \text{ Davenport.}$$
$$120 \text{ Davenport.}$$

OPERATION.

$$\frac{7 \times 10 \times 21 \times 120}{8 \times 14 \times 25} = \frac{1764}{2800} = 630 \text{ bushels, Ans.}$$

(7.)

STATEMENT.

$$24s. \text{ Massachusetts} = \ 32s. \text{ New York.}$$
$$48s. \text{ New York } = \ 45s. \text{ Pennsylvania.}$$
$$15s. \text{ Pennsylvania } = \ 10s. \text{ Canada.}$$
$$100s. \text{ Massachusetts.}$$

OPERATION.

$$\frac{32 \times 45 \times 10 \times 100 = 1440000}{24 \times 48 \times 15 =} \overline{1728} = 83\frac{1}{3}s., \text{ Ans.}$$

MISCELLANEOUS EXAMPLES IN PROPORTION.

(Page **248.**)

(1.)

BY ANALYSIS.

$27 \times 7 = 189 =$ miles A is ahead of **B**.

$36 - 27 = 9$ miles that B gains each day on A. If, therefore, 9 miles are gained in òne day, it will require, to gain 189 miles, $189 \div 9 = 21$ days, Ans.

FORM OF STATEMENT.

$36 - 27 = 9$m. : 189m. : : 1 day : 21 days, Ans.

(2.)

BY ANALYSIS.

2s. 3d. $= 27$d., price obtained for the coffee. It is evident that 27d. is $\frac{135}{100}$ of the cost; therefore, $\frac{100}{135}$ of 27d. $= 20$d. was the cost, Ans.

FORM OF STATEMENT.

135d. : 100d. : : 27d. : 20d., Ans.

(3.)

FORM OF STATEMENT.

$2000 \times 12 \times 7 \times 14 = 2352000 \div 16 = 147000$lb. whole quantity.

$105 \times 200 = 21000$lb. wholly spoiled.

$147000 - 21000 = 126000$lb. left to subsist on.

$2000 \times 12 \times 7 = 168000$ rations.

$2016000 \div 168000 = 12$oz. for each man per day, Ans.

(4.)

FORM OF STATEMENT.

$2000 \times 12 \times 7 \times 12 = 2016000 \div 16 = 126000$lb. quantity subsisted on.

$126000 \div 6 = 21000$lb. spoiled.

$21000 \times 7 = 147000$lb., the whole quantity, Ans.

(5.)

FORM OF STATEMENT.

$2000 \times 12 \times 7 \times 14 = 2352000 \div 16 = 147000$lb., whole
weight.

$2000 \times 12 \times 7 \times 12 = 2016000 \div 16 = 126000$lb. left to
subsist on, Ans.

(6.)

FORM OF STATEMENT.

$3.00 \times 60 = \$180$, price given for the Holland.
$4.00 \times 60 = \$240$, price obtained for it.
$180 : \$240 : : \$240 : \$320$, Ans.

7. $1\frac{1}{2}$lb. : 20lb. : : $1 : \$13.33\frac{1}{3}$.
   15lb. : 62lb. : : $12 : \$49.60$.
   $13.33\frac{1}{3} + \$49.60 = \$62.93\frac{1}{3}$, price of the tea.
   20lb. $+$ 62lb. $=$ 82lb., whole quantity of the tea.
   3lb. : 82lb. : : $4 : \$109.33\frac{1}{3}$; $109.33\frac{1}{3} - \$62.93\frac{1}{3} =$

8. 2 fur. 3rd. 3yd. $=$ 1824 feet.   [$46.40, Ans.
   66ft. : 64ft. : : 1824ft. : $1768\frac{8}{11}$ft.
   $1768\frac{8}{11}$ft. $\times 4 = 7074\frac{10}{11}$ feet, Ans.

9. 7 cows : 3 cows : : 5 oxen. : $2\frac{1}{7}$ oxen.
   $2 + 2\frac{1}{7} = 4\frac{1}{7}$ oxen; $4\frac{1}{7}$ oxen : 5 oxen : : 87 days : 105
   days, Ans.

10. 8mo. : $6 - 4 = 2$mo. : : 360 men : 90 men.
    $360 - 90 = 270$ men, Ans.

11. $10\frac{1}{4} = 10.25$; $1\frac{7}{8} = 1.875$; $100 - 5 = 95$; $100 : 95$
    : : 1.875yd. : 1.78125yd.; $100 : 95$: : 1.78125yd. :
    1.6921875yd.; $1.6921875 : 1 : : 10.25$yd. : $6\frac{182}{313}$yd.,
    Ans.

12. 130A. 2R. 20p. $=$ 20900p.; $20900 \times 4 = 83600$;
    100A. 0R. 30p. $=$ 16030p.; $16030 \times 5 = 80150$;
    83600p. : 80150p. : : $6537.50 : \$6267.71+$, Ans.

13. 192 tons 17cwt. 16lb. $=$ 385716lb.; $800 + 101 + 56 +$
    $43 = 1000$.
    1000 parts : 800 parts : : 385716lb. : $308572\frac{4}{5}$lb., copper.

1000 parts : 101 parts : : 385716lb. : 38957$\frac{7.9}{2.5.0}$lb. tin.

1000 parts : 56 parts : : 385716lb. : 21600$\frac{1.2}{1.2.5}$lb. zinc.

1000 parts : 43 parts : : 385716lb. : 16585$\frac{1.9.7}{2.5.0}$lb. lead.

14. 81 w. pine : 42 w. pine : : 1 oak : $\frac{4.2}{8.1}$ oak.

   7 : 8 : : $\frac{4.2}{8.1}$: $\frac{4.8}{8.1}$; that is, oak is to pitch pine as 48 to 81; $\frac{4.2}{8.1} + \frac{4.8}{8.1} = \frac{9.0}{8.1}$; that is, $\frac{9.0}{8.1}$ cords of oak are equal to 2 cords of pine; therefore, $\frac{9.0}{8.1}$ oaks : 1 oak : : 2 pine : 36 pine.

   $36 \div 2 = 18$ cords of each, Ans.

15. $63 \times \frac{4}{5} = 50\frac{2}{5}$ gallons.

   85gal. : $50\frac{2}{5}$: : $116.95 : 69.34\frac{3.8}{4.2.5}$, Ans.

16. 4 cows : 15 cows : : 3 horses : $11\frac{1}{4}$ horses.

   1 sheep : 7 sheep : : $\frac{1}{3}$ cow : $\frac{7}{3}$ cows.

   1 cow : $\frac{7}{3}$ cows : : $\frac{3}{4}$ horse : $1\frac{3}{4}$ horses.

   $8 + 11\frac{1}{4} + 1\frac{3}{4} = 21$ horses.

   Then as 21 horses : 8 horses $\Big\}$
            $3\frac{1}{2}$tons : $4\frac{9}{10}$ tons $\Big\}$: : 30 days : 16 days, Ans.

17. $25 : 14 : : 1 : \frac{14}{25}$.

   $7 : 5 : : 1 : \frac{5}{7}$.

   $5 : 2 : : 1 : \frac{2}{5}$.

   $\frac{14}{25} \times \frac{5}{7} \times \frac{2}{5} = \frac{140}{875} = \frac{4}{25}$; $2000 \div \frac{4}{25} = 12500$ pounds of turnips, are equal to 2000 pounds of potatoes; $14 : 25 : : 35$ cts. for beets : $62\frac{1}{2}$ cts., the proportionate price of potatoes; that is, 80 cts. for potatoes is as much dearer than 35 cts. for beets, as 80 is more than $62\frac{1}{2} = 17\frac{1}{2}$ cts.; again, 5 : 2 : : 25 cts. for carrots : 10 cts., the proportionate price for turnips; but 20 cts. for turnips is 10 cts. dearer than the proportionate price; hence, 20 cts. for turnips is 10 cts. dearer than 25 cts. for carrots, Ans.

18. A travels $22\frac{1}{2}$ days at the rate of 18 miles per hour $= 405$ miles. He not only travels as far as B, but the distance that B would travel in twice 9 days $= 18$ days; therefore, B, to travel the whole distance which A has travelled, would require $22\frac{1}{2} + 18 = 40\frac{1}{2}$ days; therefore, $405 \div 40\frac{1}{2} = 10$ miles per day, Ans.

19. By the conditions of the question, 2 men bear the expenses of the ride for 20 miles, 3 men for 52 miles, 4 men for 42 miles, and 5 men for 30 miles; therefore, each of the "two men" will pay $\frac{1}{2}$ the hire for 20 miles, $+\frac{1}{3}$ for 52 miles, $+\frac{1}{4}$ for 42 miles, $+\frac{1}{5}$ for 30 miles, $=$ the hire for $43\frac{5}{6}$ miles.

The "two men" will pay for $\quad 43\frac{5}{6} \times 2 =$ hire for $87\frac{2}{3}$ miles.
A will pay $\frac{1}{3}$ for 52, $+\frac{1}{4}$ for 42, $+\frac{1}{5}$ for 30 $= \quad$ " $\quad 33\frac{5}{6}$ miles.
B will pay $\qquad \frac{1}{4}$ for 42, $+\frac{1}{5}$ for 30 $= \quad$ " $\quad 16\frac{1}{2}$ miles.
C will pay $\qquad \bullet \qquad \frac{1}{5}$ for 30 $= \quad$ " $\quad \underline{\quad 6}$ miles.
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad 144$

$144 : 87\frac{2}{3} : : \$ 25 : \$ 7.609\frac{133}{108}$, share of each of the "two men."
$144 : 33\frac{5}{6} : : \$ 25 : \$ 5.873\frac{91}{108}$, A's share.
$144 : 16\frac{1}{2} : : \$ 25 : \$ 2.864\frac{7}{12}$, B's share.
$144 : \ \ 6 \ \ : : \$ 25 : \$ 1.041\frac{2}{3}$, C's share.

PERCENTAGE.

(ART. **346,** p. **252.**)

1.	.19	6.		.77$\frac{1}{2}$
2.	.27	7.		1.06
3.	.135	8.		1.07
4.	.0175	9.		8.05
5.	.074	10.		9.998

ART. **347,** p. **252.**)

2.	75bu.	8.		$990
3.	15cwt.	9.		48bbl.
4.	45 tons.	10.		60hhd.
5.	$ 375	11.		$ 0.25
6.	665chal.	12.		15.12lb.
7.	.8 miles.			

∫

(13.)	(14.)
900	$ 1728
.08	.15
72.00	8640
	1728
900	$ 259.20
72	
828	$ 1728
.50	259.20
414.00, Ans.	$ 1468.80, Ans.

(15.)

$ 25000		$ 25000
.40		10000
$ 10000.00 = wife's share.		$ 15000

$ 15000		$ 10000
.30		4500
$ 4500.00 = son's share.		$ 14500

$$\begin{array}{r} \$ 25000 \\ 14500 \\ \hline 10500 \\ 60 \\ \hline 3)10440 \\ \hline \$ 3480 = \text{each daughter's share.} \end{array}$$

Ans. Wife, $ 10000; son, $ 4500; each daughter, $ 3480.

(ART. 348, p. 253.)

2.	$12\frac{1}{2}$ per cent.	7.	75 per cent.
3.	10 per cent.	8.	$37\frac{1}{2}$ per cent.
4.	$5\frac{3}{8}$ per cent.	9.	50 per cent.
5.	25 per cent.	10.	20 per cent.
6.	$ 36.00 × .25 = $ 9.00 ;	11.	20 per cent.
	$ 9.00 ÷ 150 = .06, Ans.		

conditions
the ride for
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wo men " w
iles, $+ \frac{1}{4}$ for
$3\frac{5}{8}$ miles.

o men " will
y $\frac{1}{3}$ for 52, $-$
ay
ay

$7\frac{2}{3} : : \$ 25 : \$$
$3\frac{5}{8} : : \$ 25 : \$$
$3\frac{1}{2} : : \$ 25 : \$$
$3 \quad : : \$ 25 : \$$

10. 6yd. $1\frac{1}{10}$qr. $\div$ .25 $=$ 25yd. $0\frac{4}{10}$qr. $=$ quantity cut off;
    25yd. $0\frac{4}{10}$qr. $+$ 6yd. $1\frac{1}{10}$qr. $=$ 31yd. 1qr. 2na., Ans.

11. 279 $\div$ .90 $=$ 310, taken; 310 $-$ 279 $=$ 31, lost, Ans. .

12. 100 $-$ $86\frac{1}{2}$ $=$ $13\frac{1}{2}$ per cent., which A had left.
    100 $-$ 55 $=$ 45 per cent., which B had left.
    Hence, $.13\frac{1}{2}$ $+$ .45 $=$ $.58\frac{1}{2}$ which they both have left of 200 $=$

    $$\$ 36.85\frac{1}{2} ; \quad \frac{\$ 36.85\frac{1}{2} \times 200}{58\frac{1}{2}} = \$ 63 ;$$

    100 : $13\frac{1}{2}$ :: \$63 : $\$ 8.50\frac{1}{2}$, what A had left.
    100 : 45 :: \$63 : \$ 28.35, what B had left.

MISCELLANEOUS EXAMPLES.

(Page **257**.)

1.	$117\frac{1}{25}$	7.	.07
2.	$174\frac{14}{15}$	8.	$81\frac{1}{4}$.
3.	351	9.	\$ 66.
4.	$2\frac{7}{10}$	10.	$5\frac{35}{99}$.
5.	$316\frac{2}{3}$	11.	\$ 22.
6.	40	12.	$117\frac{8}{9}\frac{1}{4}$lb.

13. 6 $\times$ 30 $\times$ 12 $=$ 2160; 2160 $\times$ .15 $=$ 324, Ans.

14. 12 : 100 :: \$ 0.69 : \$ 5.75, Ans.

15. 5400 $+$ 6000 $=$ what was excavated in the first two weeks
    $=$ 11400; 40500 $-$ 11400 $=$ 29100; 29100 $\times$ .25 $=$
    7275 $=$ what was excavated in the third week; 11400
    $+$ 7275 $=$ 18675; 40500 $\div$ 2 $=$ 20250; 20250 $-$
    18675 $=$ 1575 cubic feet, Ans.

16. $.25 \times .50 = .12\frac{1}{2}$; $\dfrac{.12\frac{1}{2} \times 100}{75} = .16\frac{2}{3}$, Ans.

17. \$ 0.50 $\times$ .80 $=$ \$ 0.40, the cost of a gallon; \$ 0.40 $\times$ .25
    $=$ \$ 0.10; \$ 0.40 $+$ \$ 0.10 $=$ \$ 0.50, Ans.

18. 20 $\times$ .04 $=$ .8; 1 $\times$ .05 $=$ .05; 20 $-$ .8 $=$ 19.2 $=$ the
    length after shrinking; 1 $-$ .05 $=$ .95 $=$ width after
    shrinking; 19,2 $\times$ .95 $=$ $18\frac{6}{25}$; 20 $-$ $18\frac{6}{25}$ $=$ $1\frac{19}{25}$yd.,
    Ans.

19. 15 per cent. of 100 $\quad= .15 \qquad=$ daughter's;

 10 per cent. of .85 $\quad= .085 \quad + \frac{1}{10}$ of an acre = son's;

 25 per cent. of .765 $- \frac{1}{10} = .19125 - \frac{1}{10}$ of an acre = wife's;

        $.42625 + \frac{3}{40} =$ the shares of

the wife, the son, and the daughter; and $100 - .42625$
$=$ the remainder $+ \frac{3}{40}$ of an acre which is not included in
the .42625. Hence $100 - .42625 = .57375 = 39 \times 5$
$= 195 + \frac{3}{40}$ of an acre $= 195\frac{3}{40}$ acres.

 $.57375 : 100 :: 195\frac{3}{40}$ acres : 340 acres, Ans.

20. $30,500,000 \times 1.34\frac{1}{2} = 41,022,500$, Ans.

21. Let the English $= 100$ per cent.;

 $100 + 33\frac{1}{3} = 133\frac{1}{3}$ per cent. $=$ French;

 $133\frac{1}{3} + 8\frac{1}{3}$ of $133\frac{1}{3} = 144\frac{4}{9}$ per cent. $=$ Turks;

 $144\frac{4}{9} - 100 = 44\frac{4}{9}$ per cent. $= 1600$. Hence

 $44\frac{4}{9} : 100 \quad:: 1600 : 3600 =$ English;

 $100 \;: 133\frac{1}{3} :: 3600 : 4800 =$ French;

 $100 \;: 144\frac{4}{9} :: 3600 : \underline{5200} =$ Turks;

       13600 the whole number.

 $13600 : 3600 :: 100 : 26\frac{8}{17}$, percentage of English;

 $13600 : 4800 :: 100 : 35\frac{5}{17}$, percentage of French;

 $13600 : 5200 :: 100 : 38\frac{4}{17}$, percentage of Turks.

22. $\$7.25 \times .10 = \$0.725$; $\$7.25 - \$0.725 = \$6.525$;

 $\$7.25 + \$0.725 = \$7.975$; $\$7.975 - \$6.525 =$
$\$1.45$; $\$6.525 : \$1.45 :: 100 : 22\frac{58}{261} =$ the per cent.
of the proceeds of the flour above the cost; $560 \times \$1.45$
$= \$812.00 =$ profits, Ans.

23. $87500 \div 1.25 = 70000$, Ans.

24. Let the cost of the horse $= 100$; but the horse cost $62\frac{1}{2}$ per
cent. as much as the buggy; hence $62\frac{1}{2} : 100 :: 100 :$
$160 =$ the proportionate percentage of the buggy; $100 -$
$70 = 30$, the proportionate percentage of the harness;
$100 + 160 + 30 = 290$;

 $290 : 100 :: \$500 : \$172\frac{12}{29} =$ horse;

 $290 : 160 :: \$500 : \$275\frac{25}{29} =$ buggy;

 $290 : \;\,30 :: \$500 : \$ \;51\frac{21}{29} =$ harness.

INTEREST.

(ART. 354, p. 262.)

2.	$40.50
3.	$476.144
4.	$161.33
5.	$888.546
6.	$108.587
7.	$44.442
8.	$540.000
9.	$1.30
10.	$1.275
11.	$0.6867
12.	$31.85
13.	$116.99
14.	$40.10
15.	$0.0758
16.	$0.9739
17.	$14.607
18.	$0.227
19.	$44.034
20.	$18.674
21.	$193.09
22.	$3.37½
23.	$26.343
24.	$0.584
25.	$17.438

26.

	y.	mo.	da.
	1863	6	27
	1852	0	11
	11	6	16

$76.895 × .692⅔ = $53-
[.262, Ans.

27.

	y.	mo.	da.
	1859	9	19
	1856	6	4
	3	3	15

$98.25 × .197½ = $19-
[.404, Ans.

28.

	y.	mo.	da.
	1860	6	18
	1836	1	19
	24	4	29

$22.763 × 1.464⅚ = $33-
[.344, Ans.

29.

	y.	mo.	da.
	1859	9	12
	1855	0	7
	4	9	5

$175.07 × .285⅚ = $50-
[.04, Ans.

30.

	y.	mo.	da.
	1854	0	11
	1852	11	6
	1	1	5

$197.285 × .065⅚ = $12-
[.987, Ans.

31. $4377.15 × .18 = $787-
.887; $787.887 + $43-
77.15 = $5165.037 =
the amount, Ans.

32. $444.60 × .33 = $146-
.718, Ans.

(ART. **355**, p. **265**.)

(2.)

$$\begin{array}{r} \$16.75 \\ .07 \\ \hline \end{array}$$

Interest for 1 year	$=\overline{1.1725}$
Int. for 4 mo., $\frac{1}{3}$ of a year	$=\overline{.3908}$
Int. for 2 mo., $\frac{1}{2}$ of 4 mo.	$=.1954$
Int. for 1 mo., $\frac{1}{2}$ of 2 mo.	$=.0977$
Int. for 15 da., $\frac{1}{2}$ of 1 mo.	$=.0488$
Int. for 2 da., $\frac{1}{15}$ of 1 mo.	$=.0065$
Int. for 7 mo. and 17 da.	$=\$\overline{0.7392}$, Ans.

(3.)

y.	mo.	d.
1852	11	7
1852	3	17
	7	20

$$\begin{array}{r} \$11.105 \\ .07 \\ \hline \end{array}$$

Int. for 1 year	$=\overline{.77735}$
Int for 6 mo., or $\frac{1}{2}$ of 1 yr.	$=\overline{.38867}$
Int. for 1 mo., or $\frac{1}{6}$ of 6 mo.	$=.06477$
Int. for 15 da., or $\frac{1}{2}$ of 1 mo.	$=.03238$
Int. for 5 da., or $\frac{1}{3}$ of 15 da.	$=.01079$
Int. for 7 mo. and 20 da.	$=\$\overline{0.49661}$, Ans.

	y.	mo.	d.				y.	mo.	d.
4.	1854	7	30		8.		1860	0	11
	1853	0	2				1852	1	7
		1	7	28				7	11	4

$12.69 \times .099\frac{2}{3} = \$1.2647$; $17869.75 \times .475\frac{2}{3} = \$8500.$
$\quad\$1.2647 \div 6 = \$0.2107,$ $.0444$; $\$8500.0444 \div 6$
$\quad\times 7 = \$1.475$, Ans. $= \$1416.67406, \times 5 =$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\$7083.3703$, Ans.

5.		$746.666
6.		$504.64
7.		$0.70

9. $21.78

10. $0.149

11.
	y.	mo.	d.
	1863	11	15
	1861	10	11
	2	1	4

$35.61×.125⅔=$4.474, Ans.

12. $10.139

13.
	y.	mo.	d.
	1860	6	17
	1854	4	7
	6	2	10

$1728.19
.371⅔

6 × 4 = 24)642.31061⅔
$26.762,
[Ans.

14. $397.16
.05½
198580
19858

Int. for 1 yr. = 21.8438
Int. for ½ of 1 yr. = 10.9219
Int. for 1 da., or
¹⁄₁₈₀ of 6 mo. = .0608
Ans. $32.8265

15. $100.25
.014⅚

At 6 per cent. = 1.48704½
At 1 per cent. = .24784
4
At 4 per cent. = .99136
$100.25
Amount, $101.241
12*

16. $51.17
.04
2)$2.0468

Int. for 6 mo. = 1.0234
Int. for 3 mo. = .5117
Int. for 15 da., or ⅙ of
3 mo. = .08528
Int. for 1 da. = .00568
Int. for 13 da. = .07384
Ans. 1.6999

17. $42.20 × .062⅔ = $2.6-
445; 4½ × $2.6445 ÷ 6
= 1.983, Ans.

18. $0.975

19.
	y.	mo.	d.
	1852	6	3
	1849	10	27
	2	7	6

$96.84
.07½
67788
4842

Int. for 1 yr. = 7.2630
2
Int. for 2 yr. = 14.5260
Int. for 6 mo. = 3.6315
Int. for 1 mo. = .6052
Int. for 6 da., or
⅕ of 1 mo. = .1210
Ans. $18.8837

20. $225.925

	y.	mo.	d.
21.	1857	5	19
	1853	6	29
	3	10	20

$71.09,1
.233⅓
———
213273
213273
142182
23697

Interest at 6 per cent. } $16.587900
2

Ans. $33.17,5800

*The interest will be twice this amount at 12 per cent.

22. $444.163
23. $7.864
24. $0.886

	y.	mo.	d.
25.	1857	1	18
	1829	4	16
	27	9	2

$36.72 × 1.665⅓ = $61.151-04; 7 × $61.15104 ÷ 6 = $71.342, Ans.

26. $8.628
27. $515.60
28. $3.167
29. $9750 × .08 = $780.00, [Ans.
30. $9162×3×1½=$412.29, [Ans.
31. $95.833
32. $58.00
33. $246.295

34. $10000
.15⅓
———
$150000
3333⅓

Int. at 6 per cent. = $1533.33⅓
7
———
6)10733.33⅓

Int. at 7 per cent. = $1788.88

$1788.88 − $1533.33 = $2-55.55, Ans.

(ART. 356, p. 268.)

2. $96.00
Int. for 60 da. = .96
Int. for 30 da. = .48
Int. for 3 da. = .048
———
6).528
.088
7
———
Int. at 7 per cent. = $0.616 [Ans.

3. $320.40
Int. for 60 da. = $\frac{1}{100}$ of the principal = 3.204
Int. for 3 da. = $\frac{1}{20}$ of 60 da. = .1602
———
6)3.3642
.5607
5
———
Int. at 5 per cent. = $2.8035 [Ans.

4.	$131.20	15.	$144.50

Int. of 60 da. = 1.3120
Int. of 60 da. = 1.3120
Int. of 3 da.   = .0656
Ans. $2.6896

$144.50
.024
6)3.46800
578
5
73)2.890
39
$2.851 = $2.89
[diminished* by $\frac{1}{73}$ of itself,
[Ans.

5. $0.345
6. $245.65
7. $1.681
8. $0.263
9. 177.029\frac{13}{18}$
10. 166.681\frac{2}{3}$ 16. $20.662
11. $569.70   17. $19.65
12. $35.295   18. $80.38
13. $72.888   19. $1005.50
14. $2.223   20. $7321.90

* See note 2, p. 267.

(ART. **357**, p. **269**.)

2. 179£. 12s. 11d. = 179.645£. × .095 = 17.0662£. = interest at 6 per cent.; 17.0662£. ÷ 6 = 2.8443 × 5 = 14.2215£. = 14£. 4s. 5¼d., Ans.

(3.)

Principal   = 25£.
$\frac{1}{10}$ of the principal   = 2.5   int. for 20 mo.
$\frac{1}{20}$ of int. for 1yr. 8mo. = .125   int. for 1 mo.
2.625 ÷ 6 = .4375 × 5 = 2.1875£.
= 2£. 3s. 3¾d. = int. for 1yr. 9mo., Ans.

(4.)

Principal   = 5440.5£.
$\frac{1}{5}$ of the principal   = 1088.1   int. for 3yr. 4mo.
$\frac{1}{8}$ of int. for 3yr. 4mo. = 136.0125 int. for 5mo.
$\frac{2}{5}$ of int. for 5 mo.   = { 27.2025 } int. for 2mo.
{ 27.2025 }
1278.5175£. (Carried forward.)

(Brought forward:) 1278.5175£. = the interest for 3yr. 11mo. = 1278£. 10s. 4d., Ans.

5. 943£. 1s. 8d. = 943.0833£.; 30da. in May + 30 in June + 31 in July + 31 in Aug. + 30 in Sept. + 21 in Oct. = 173da.; 943.0833£. × .028⅚ = 27.19223£.; 5¼ × 27.19223£. ÷ 6 = 23.793£.; 23.793 − .3259 (₁⁄₇₃ of 23.793£.) = 23.467 = 23£. 9s. 4d., Ans.

(ART. 358, p. 270.,

2. $10.08 ÷ .07 = $144, Ans.

3. $1.00×.08¼ = .0825×1⅓ = .11; 13.20÷.11 = $120.00, Ans.

4. $40.50 ÷ .06 = $675, Ans.

5. $1.00 × .02 × 3 = .06; $24.00 ÷ .06 = $400, Ans.

6. $1.00×.0305 = $0.0305÷6 = $.0050⅚×7¼ = .0368½⅓; $206.38¼ ÷ .0368⁴⁴⁄ = $5600, Ans.

(ART. 359, p. 270.)

2. $144.00 × .01 = $1.44; 10.08 ÷ 1.44 = 7 per cent., Ans.

3. $120 × .01 = 1.20 × 1⅓ = 1.60; 133.20 − 120 = 13.20 ÷ 1.60 = 8¼ per cent., Ans.

4. $1.00 × .01 = $0.01 × 14⁴⁄₇ = $0.14⁴⁄₇; 1.00 ÷ 14⁴⁄₇ = 7 per cent., Ans.

5. $1.00 × .01 = $.01 × 33⅓ = $.33⅓; 3.00 ÷ .33⅓ = 9 per cent., Ans.

6. $4650 × .01 = $46.50; 232.50 ÷ 46.50 = 5 per cent., Ans.

7. 48da. = 1⅗mo. = $\frac{1⅗}{12}$ of 1yr.; $7500 × .01 = $75;

$75 × $\frac{1⅗}{12}$ = $10; 60 ÷ 10 = 6 per cent., Ans.

8. $280 × .01 = $2.80 × 6½ = $18.20; $411.95 — $280 = $131.95; 131.95 ÷ 18.20 = 7¼ per cent., Ans.

9. $480 × .01 = $4.80 × 1$\frac{7}{24}$ = $6.20; $529.60 — $480 = $49.60; 49.60 ÷ 6.20 = 8 per cent., Ans.

(ART. 360, p. 271.)

2. $120 × .08¼ = $9.90; 13.20 ÷ 9.90 = 1⅓yr., Ans.

3. $144 × .07 = $10.08; 10.08 ÷ 10.08 = 1yr., Ans.

4. $240 × .06 = $14.40; $280—$240 = $40; 40÷14.40 = 2⅞yr. = 2yr. 9mo. 10da., Ans.

5. $1.00 × .05 = $.05; 1.00 ÷ .05 = 20yr., Ans.

6. $1.00 × .10 = $.10; 1.00 ÷ .10 = 10yr., Ans.

7. $1500 × .05 = $75.00; $2250 — $1500 = $750; 750 ÷ 75 = 10yr., Ans.

8. $480 × .04½ = $21.60; $561.60 — $480 = $81.60; 81.60 ÷ 21.60 = 3⅞yr. = 3yr. 9mo. 10da., Ans.

9. $1728 × 12 = $207.36; $3853.44 — $1728 = $2125.44; 2125.44 ÷ 207.36 = 10¼yr. = 10yr. 3mo., Ans.

10. $240 × .06 = $14.40; $720 — $240 = $480; 480 ÷ 14.40 = 33⅓yr. = 33yr. 4mo., Ans.

11. $400 × .06 = $24.00; 100 ÷ 24 = 4⅙yr. = 4yr. 2mo.;

```
1857   4   16
   4   2    0
─────────────
1861   6   16  = July 16, 1861, Ans.
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PROMISSORY NOTES.

(PAGE 275.)

(2.)

Principal on interest from June 17, 1849, . .	$769.870
Interest from June 17, 1849, to March 1, 1850, 8mo. 14da., ...	32.591
Amount (carried forward), . . .	$802.461

(Brought forward :)	$ 802.461
First payment, March 1, 1850, ...	75.500
New principal, bearing interest from March 1, 1850,	726.961
Interest from March 1, 1850, to June 1d, 1851,	
15mo. 10da., ...	55.733
Amount, ...	782.694
Second payment, June 11, 1851, ...	165.000
New principal, bearing interest from June 11, 1851,	617.694
Interest from June 11, 1851, to Sept. 15, 1851,	
3mo. 4da., ...	9.677
Amount, ...	627.371
Third payment, Sept. 15, 1851, ...	161.000
New principal, bearing interest from Sept. 15, 1851,	466.371
Interest from Sept. 15, 1851, to Jan. 21, 1852,	
4mo. 6da., ...	9.793
Amount, ...	476.164
Fourth payment, January 21, 1852, . . .	47.250
New principal, bearing interest from Jan. 21, 1852,	428.914
Interest from Jan. 21, 1852, to December 6, 1853,	
22mo. 15da., ...	48.252
Amount, ...	477.166

Fifth payment, less than interest, March 5, 1853, ... 12.17

Sixth payment, more than interest, Dec. 6, 1853, ... 98.00

	110.170
New principal, bearing interest from Dec. 6, 1853,	366.996
Interest from Dec. 6, 1853, to July 7, 1854, 7mo. 1d.,	12.906
Amount (carried forward), . . :	$ 379.902

(Brought forward:) $379.902

Seventh payment, July 7, 1854, ... 169.000

New principal, bearing interest from July 7, 1854,   210.902

Interest from July 7, 1854, to Sept. 25, 1855, 14mo. 18da., ... 15.395

Balance due Sept. 25, 1855, ... $226.297

(3.)

Principal on interest from April 30, 1851,   .   . $300.000

Interest from April 30, 1851, to June 27, 1852, 13mo. 27da., ... 20.850

Amount, ... 320.850

First payment, June 27, 1852,   .   .   . 150.000

New principal, bearing interest from June 27, 1852,   170.850

Interest from June 27, 1852, to December 9, 1852, 5mo. 12da., ... 4.612

Amount, ... 175.462

Second payment, Dec. 9, 1852,   .   .   . 150.000

New principal, bearing interest from Dec. 9, 1852,   25.462

Interest from Dec. 9, 1852, to Oct. 9, 1853, 10mo.,   1.273

Balance due Oct. 9, 1853, ... $26.735

(4.)

Principal on interest from Feb. 11, 1852,   .   . $54.180

Interest from Feb. 11, 1852, to July 11, 1853, 17mo.,   4.605

Amount, ... 58.785

First payment, July 11, 1853,   .   .   . 12.250

Principal bearing interest from July 11, 1853,   . 46.535

Interest from July 11, 1853, to Aug. 21, 1855, 25mo. 10da., ... 5.894

Amount (carried forward),   .   .   . $52.429

(Brought forward :)		$ 52.429
Second payment, less than interest, Aug. 15, 1854, ...ˈ. .	2.100	
Third payment, less than interest, July 9, 1855, ...	3.120	
Fourth payment, more than interest, Aug. 21, 1855, ...	37.180	
		42.400
Principal bearing interest from Aug. 21, 1855, .		10.029
Interest from Aug. 21, 1855, to Dec. 17, 1855, 3mo. 26da., ...		.193
Balance due Dec. 17, 1855, ...		$ 10.222

(6.)

Principal carrying interest from Jan. 1, 1850, .		$ 1000.00
Interest from Jan. 1, 1850, to Sept. 28, 1850, 8mo. 27da., ...		51.91
Amount, ...		1051.91
First payment, ...		144.00
Balance for new principal, ...		907.91
Interest from Sept. 28, 1850, to July 17, 1851, 9mo. 18da., ...		51.01
Amount, ...		958.92
Second payment, March 1, 1851, a sum less than interest, ...	20.00	
Third payment, July 17, 1851, a sum greater than interest, ...	360.00	
		380.00
Balance for new principal, ...		578.92
Interest from July 17, 1851, to Aug. 9, 1851, 20da.,		2.47
Amount (carried forward),		$ 581.39

(Brought forward :)	$581.39
Fourth payment, Aug. 9, 1851, ...	190.00
Balance for new principal, ...	391.39
Interest from Aug. 9, 1851, to Sept. 25, 1852, 1yr. 1mo. 16da., ...	30.89
Amount, ...	422.28
Fifth payment, Sept. 25, 1852, ...	170.00
Balance for new principal, ...	252.28
Interest from Sept. 25; 1852, to Dec. 11, 1853, 1yr. 2mo. 16da., ...	21.38
Amount, ...	273.66
Sixth payment, Dec. 11, 1853, ...	200.00
Balance for new principal, ...	73.66
Interest from Dec. 11, 1853, to July 4, 1855, 1yr. 6mo. 23da., ...	8.06
Amount, ...	81.72
Seventh payment, July 4, 1855, ...	75.00
Balance for new principal, ...	6.72
Interest from July 4, 1855, to June 1, 1857, 1yr. 10mo. 27da., ...	.89
Balance due at the time of payment, . . .	$7.61

NOTE. — Interest on the above note is computed at 7 per cent.

(ART. 372, p. 278.)

(2.)

Principal, ...		$700.00
Interest for 9mo. 24da., ...		34.30
		734.30
First payment, ...	160.00	
Interest for 8mo. 10da., ...	6.66⅔	
(Carried forward :)	$166.66⅔	$734.30

13

(Brought forward :)	$166.66⅔	$734.30
Second payment, ...	200.	
Interest for 5mo. 4da., ...	5.13⅓	
Third payment, ...	120.	
Interest for 2mo. 17da., . . .	1.54	
Fourth payment, ...	60.	
Interest for 1mo. 23da., . . .	.53	
		553.87
Balance due Nov. 28, 1854, . .		$180.43

(3.)

Principal, ...		$500.00
Interest for 12mo., ...		35.00
		535.00
Payment, ...	200.00	
Interest for 9mo., ...	10.50	
		$210.50
Balance due April 1, 1858, . .		$324.50

COMPOUND INTEREST.

(ART. 376, p. 280.)

(2.)

$500	(Brought up :) 31.80
.06	530
30.00	561.80
·500	.06
530	33.7080
.06	561.80
81.80 (Carried up.)	$595.5080, Ans.

(3.)	(4.)
$970	$300
.06	.07
58.20	21.00
970	300
1028.20	321
.06	.07
61.6920	22.47
1028.20	321
1089.8920	343.47

.049 Int. of $1 for 9mo. 24da.   .07

(3.)	(4.)
98090280	24.0429
43595680	343.47
53.404708	367.5129
1089.8920	.07
1143.2967	25.725903
970   First principal subtracted.   367.5129
$173.296, Ans.	393.2388
	.035
	19661940
	11797164
	13.763358
	393.238
	407.001
	300
	$107.001, Ans.

(5.)

$316	(Brought up.)	376.361
.06		.023

$316
.06
―――
18.96
316
―――
334.96
.06
―――
20.0976
334.96
―――
355.0576
.06
―――
21.303456
355.0576
―――
376.361 (Carried up.)

(Brought up.) 376.361
.023
―――
1129083
752722
―――
8.656303
376.361
―――
385.017
316
―――
$ 69.017, Ans.

(Art. 377, p. 282.)

2. $1200 × 2.104852 = 2525.8224; 2525.8224 — 1200 = $1325.822, Ans.

3. $300 × 1.790848 = 537.2544; 537.2544 × 1.0375 = 557.401; 557.401 — 300 = $257.401, Ans.

4. $5 × 29.457 = $147.285; $147.285 — $5 = $142.285, Ans.

5. $480 × 10.285718 = $4937.144, Ans.

6. $40 × .310796 = $12.431, Ans.

7. The amount of $1 for 50 years is $18.420154; if now this amount be taken for a principal and multiplied by the amount of $1 for 50 years, the result will be the amount of $1 for 100 years; therefore 18.420154 × 18.420154 = 339.302072, the amount of $1 for 100 years; and 339.302072 × $100 = $33930.2072; and $33930.2072 — $100 = $33830.2072, Ans.

8. $1000 × 6.84059 = $6840.59; $6840.59 × 1.02 = $69-77.4018; $6977.4018—$1000 = $5977.4018 = interest at compound interest; $1000 × 2.00 = $2000 =

interest at simple interest; $ 5977.4018 — $ 2000 = $ 3977.4018, Ans.

9. For 21y., payable semi-annually, is the same as for 42y., payable annually ; $ 50 × 3.460696 = $ 173.034, Ans.

(10.)

Principal bearing interest from March 26, 1854,		$ 100.000
Interest for 1y. 5mo., ...		8.650
Amount of the principal to Aug. 25, 1855, . .		108.650
First payment, June 11, 1854, . .	50.000	
Compound interest from June 11, 1854, to Aug. 25, 1855, 14mo. 14da., . .	3.653	
Second payment, Sept. 25, 1854, . .	50.000	
Compound interest from Sept. 25, 1854, to Aug. 25, 1855, 11mo., . . .	2.750	
Amount of the endorsements, ...		106.403
Balance due Aug. 25, 1855, ...		$ 2.247

(11.)

Principal bearing interest from Jan. 1, 1850, .	.	$ 1000.00
Compound interest on $ 1000 from Jan. 1, 1850, to April 1, 1855, 5y. 3mo. 21d., . . .	.	427.09
Amount of principal to April 1, 1855, . .	.	1427.09
First payment, June 10, 1850, . .	70.00	
Compound interest from June 10, 1850, to April 1, 1855, 4y. 9mo. 21d., . .	26.94	
Second payment, Sept. 25, 1851, . .	80.00	
Compound interest from Sept. 25, 1851, to April 1, 1855, 3y. 6mo. 6da., . .	21.54	
Third payment, July 4, 1852, . . .	100.00	
Compound interest from July 4, 1852, to April 1, 1855, 2y. 8mo. 27da., . .	20.43	
Fourth payment, Nov. 11, 1853, . .	30.00	
(Carried forward :) $ 348.91		$ 1427.09

13*

| | (Brought forward:) | $ 348.91 | $ 1427.09 |

(Brought forward:) $ 348.91 $ 1427.09

Compound interest from Nov. 11, 1853, to
    April 1, 1855, 1y. 4mo. 20da.,   .   .   2.97

Fifth payment, June 5, 1854, .   .   .   50.00

Compound interest from June 5, 1854, to
    April 1, 1855, 9mo. 26da.,   .   .   2.87

Amount of endorsements, ...   404.75

Balance due April 1, 1855, ... $ 1022.34

NOTE. — The above note and endorsements are reckoned at 7 per cent.

(ART. 378, p. 284.)

2. 1.60578 × 1.035 = 1.553255 = the amount of $ 1 for 6y.
    6mo. ; $ 205.90 ÷ .5532 = $ 372.16, Ans.

3. 1.191 × 1.012 = 1.2052 = amount of $ 1 for 3y. 2mo.
    12da. ; $ 1026.54 ÷ .2052 = $ 5000, Ans.

4. 1.905 = interest of $ 1 for 31y. at 3½ per cent. ; $ 857.25
    ÷ 1.905 = $ 450, Ans.

(ART. 379, p. 285.)

2. 640.405 ÷ 400 = 1.60101, which for 12 years indicates by
    the table an interest of 4 per cent., Ans.

3. 2096.147 ÷ 2500 = .8384588 ; and in the table this is
    found to be the *interest* of $ 1 for 9 years at 7 per cent.

4. 200 ÷ 100 = 2.000000 ; by the table it is seen that at 6
    per cent. this will be the amount of $ 1 for a time some-
    where between 11 and 12 years. The amount for 11y.
    is 1.898299 ; if this, increased by $\frac{89}{100}$ of a year, be 2.,
    then the rate is 6 per cent. ; 1.898299 × 1.0534 =
    2.00000 very nearly.

5. $ 31479.70 ÷ 10000 = 3.14797 ; 3.0256, the amount of $ 1
    for 19y. at 6 per cent., is the approximation ; 3.0256 ×
    1.04⅜ = 3.14797 nearly ; hence the rate is 6 per cent.

(ART. 380, p. 285.)

2. 640.405 ÷ 400 = 1.601012 ; which is the amount of $ 1 at
    4 per cent. for 12 years, Ans.

3. 9021.78 ÷ 6000 = 1.50363, which is the amount of $1 at 7 per cent. for 6 years, Ans.

4. 200 ÷ 100 = 2.000000; by the table the amount at 5 per cent., which approximates to 2., is the amount for 14y., which is 1.979932; 2.0000 — 1.979932 = that interest which the amount for 14y. (1.979932) must gain before the $1 will be doubled; therefore, to get the time required for gaining this principal, divide (Art. 360) the given interest by the interest of 1.979932 for 1y.; that is, divide 2.000 — 1.979932 by 2.078928 — 1.979932; .020068 ÷ .098996 = .2026y. = 2mo. 13da. nearly; 14y. 2mo. 13da., Ans.

5. 200 ÷ 100 = 2.; by the table the nearest approximation to 2., at 6 per cent., is the amount for 11y., 1.898299; 2.012196 — 1.898299 = .113897; 2. — 1.898299 = .101701; .101701 ÷ .113897 = .89y. = 10mo. 20da.; 11y. 10mo. 20da., Ans.

6. 1781.665 ÷ 450 = 3.959, which by the table is found to be the amount of $1 at 3½ per cent. for 40y.; but, as the principal in the problem pays 3½ per cent. in periods of half a year, its 40 periods will be only 20y. The deposit was made when the son was 1 year old; and 20 + 1 = 21y., Ans.

DISCOUNT AND PRESENT WORTH.
(ART. 386, p. 287.)

2. $1.07 = amount of $1 for the given time; $802.50 ÷ 1.07 = $750; $802.50 — 750 = $52.50, Ans.

3. $1.12 = amount of $1 for the given time; $117.60 ÷ 1.12 = $105.00, Ans.

4. $1.205 = amount of $1 for 3y. 5mo.; $769.60 ÷ 1.205 = $638.672, Ans.

5. $1.26875 = amount of $1 for 3y. 7mo.; $678.75 ÷ 1.26875
   = $534.975, Ans.

6. $1.25 = amount of $1 for the time;
   $600 ÷ 1.25 = $480 present worth;
   $600 − $480 = $120 discount, Ans.

7. $1.056⅔ = amount of $1 for 11mo. 11da; $79.87 ÷ 1.056⅔
   = $75.57,4+.
   $1.111⅛ = amount of $1 for 22mo. 7da.; $87.75 ÷ 1.111⅛
   = $78.971 + $75.574 = $154.545+, Ans.

8. $1.118 = amount of $1 for 23mo. 18da.; $365.87 ÷ 1.118
   = $327.254+.
   $1.1525 = amount of $1 for 30mo. 15da.; $161.15 ÷ 1.1525
   = $139.826+.
   $1.028⅔ = amount of $1 for 5mo. 22da.; $112.50 ÷ 1.028⅔
   = $109.364+.
   $1.258 = amount of $1 for 51mo. 18da.; $96.81 ÷ 1.258 =
   $76.955+.
   $327.254 + $139.826 + $109.364 + $76.955+ = $653-
   .40+, Ans.

9. $67.25 ÷ 1.18 = $56.99, Ans.

10. $80.095 ÷ 1.191 = $67.25, Ans.

11. $110.364 ÷ 1.402552 = 78.687; $110.364 − $78.687 =
    $31.677 discount, Ans.

(ART. 387, p. 288.)

1. $960 × .05 = $48; $960 − $48 = $912, Ans.

2. $5000 ÷ 1.07 = $4672.89; $5000 − $4672.89 =
   $327.11 true discount; $5000 × .07 = $350 nominal
   discount; $350 − $327.11 = $22.89, Ans.

3. $4440 × .03 = 133.20; $4440 − $133.20 = $4306.80;
   $4306.80 is the nominal present worth, and is the sum
   which must be hired at 6 per cent. to pay the debt;
   $4306.80 × .03 = $129.204; $133.20 − $129.204 =
   $3.996, Ans.

BANKING.

(Art. **392**, p. **290**.)

2. $7800 $\times$ (.015 + .0005 for 3 days of grace) .0155 = $120.90, Ans.

3. $1200 $\times$ .01225 = $14.70, Ans.

4. $8000 $\times$ .0105 = $84 discount; $8000 — $84 = $7916 proceeds, Ans.

5. $760 $\times$ .0255 = $19.38; $760 — $19.38 = $740.62, Ans.

6. $7860 $\times$ .030$\frac{1}{2}$ = $239.73; $7860 — $239.73 = $76-20.27, Ans.

7. The time from Dec. 3 to April 3 is 4mo.; 4mo. from 6mo. leave 2mo. or 60da.; 60da. + 3da. = 63da., the time the note is to run; $160.40 $\times$ .0105 = $1.68; $160.40 — $1.68 = $158.72, Ans.

8. 123da. from Apr. 16, 1856 = Aug. 17, 1856, the time when the note is legally due; from May 16 to Aug. 17 = 93da., the time for which the note was discounted; $890.50 $\times$ .0155 = $13.80275; $13.80275 ÷ 6, $\times$ 7 = $16.1032, Ans.

9. 90 + 3da. from May 1 = Aug. 2, the legal time of maturity; May 1 to June 11 = 41da.; 93 — 41 = 52da., for which the note was discounted; $1340 $\times$ .008$\frac{2}{3}$ = $11.61; $1340 — $11.61 = $1328.39 proceeds, Ans.

10. From June 19 to July 5 are 16da.; from June 19 to Aug. 19 are 61da.; 61 + 3 = 64da.; 64 — 16 = 48, the time the note had to run, and for which it was discounted; $9000 $\times$ .032 = $288; $9000 — $288 = $8712, Ans.

(Art. **393**, p. **292**.)

2. $1 $\times$ .0180$\frac{5}{6}$ = 0.0180\frac{5}{6}$; $1.00 — 0.0180\frac{5}{6}$ = $0.9819, the proceeds of $1; $680 ÷ .9819 = $692.523, Ans.

3. Proceeds of $1 for 120 + 8da. = .9795; $540.50 ÷
.9795 = $551.81, Ans.

4. $1938 ÷ .9695 = $1998.968, Ans.

5. $1631.60 ÷ .9795 = $1665.74, Ans.

6. Present worth of $1 for 33da. at 2 per cent. per month =
.989; $1500 ÷ .989 = $1516.68, Ans.

(Art. 394, p. 292.)

2. The proceeds of $1 for 30 + 3 da. = .9945; .06 ÷ .9945
= $6\frac{66}{1989}$ per cent., Ans.

3. .06 ÷ .9845 = $6\frac{186}{1969}$ per cent., Ans.

4. .02 ÷ .918 = $2\frac{82}{455}$ per cent. a month, Ans.

5. The proceeds of $1 are .96442; .07 ÷ .96442 = $7\frac{12451}{48221}$
per cent., Ans.

6. .06 ÷ .94 = $6\frac{18}{47}$ per cent., Ans.

(Art. 395, p. 293.)

2. The amount of $1 for 33da. = 1.0055; .06 ÷ 1.0055 =
$5\frac{1841}{3811}$, per cent., Ans.

3. 2 per cent. a month = 24 per cent. per annum; .24 ÷ 1.042
= $23\frac{17}{521}$ per cent., Ans.

4. .06 ÷ 1.0155 = $5\frac{1841}{2831}$ per cent., Ans.

5. .08 ÷ 1.027333 = $7\frac{806669}{1027333}$ per cent., Ans.

6. .07 ÷ 1.03558 = $6\frac{39226}{51775}$ per cent., Ans.

7. .06 ÷ 1.06 = $5\frac{35}{53}$ per cent., Ans.

8. The amount of $1 for 8yr. 4mo. at 5 per cent. = $1.41\frac{2}{3}$;
.05 ÷ 1.41\frac{2}{3} = .03\frac{9}{17}, Ans.

MISCELLANEOUS EXAMPLES.

(Page 294.)

1. 18 of gold;   20 : 18 : : 100 : 90 per cent. of gold;
   1 of copper; 20 : 1 : : 100 : 5 per cent. of copper;
   1 of silver; 20 : 1 : : 100 : 5 per cent. of silver;
   20;

2. $\frac{18}{16}$ = amount, $\frac{15}{16}$ = principal, $\frac{1}{16}$ = discount. Therefore, $32 \times 16 = 512 \div 15 = 34\frac{2}{15}$qt., Ans.

3. $\$25000 \times .04 = \$1000$, Ans.

4. $206\frac{1}{4} - 192 = 14\frac{1}{4}$; $192 : 14\frac{1}{4} :: 100 : 7\frac{27}{64}$, Ans.

5. $150 : 100 :: 100 : 66\frac{2}{3}$; $100 - 66\frac{2}{3} = 33\frac{1}{3}$, Ans.

6. How long will it take $10 to double itself, or gain $10 interest? (Art. 360.) $10 \div .50 = 20$yr.; $10 \div .60 = 16\frac{2}{3}$yr.; $20 - 16\frac{2}{3} = 3\frac{1}{3}$yr., Ans.

7. $\$500 \div 1.2155 = \$411.351$, Ans.

8. $100 : 110 :: \$5 : \$5.50$, the price at which the cloth was actually sold, it being 10 per cent. less than the asking price; therefore, $90 : 100 :: \$5.50 : \$6.11\frac{1}{9}$, Ans.

9. $\$7860 \times .030\frac{1}{2} = \$239.73$; $\$7860 - \$239.73 = \$7620.27$, Ans.

10. $\$1500 \div 1.06 = \$1415.09\frac{33}{53}$; $\$1500 - \$1415.09\frac{33}{53} = \$84.90\frac{20}{53}$, the true discount; $\$1500 \times .06 = \$90$; $\$90 - \$84.90\frac{20}{53} = \$5.09\frac{33}{53}$, Ans.

11. $.24 \div .958 = 25\frac{25}{479}$ per cent., Ans.

12. Interest of $1 at $1\frac{1}{2}$ per cent. a month = $.0225$; $\$36.40 \div .0225 = \$1617.77\frac{7}{9}$, Ans.

13. $.12 \div 1.011 = 11\frac{299}{337}$ per cent., Ans.

14. In the course of 693 days the bank can discount notes of 60 days 11 times, and notes of 30 days 21 times; then (Art. 393) $\$50000 \div .9945 = \$50276.52$ = the face of the note or notes discounted for 33 days; $\$50276.52 - \$50000 = \$276.52$ = the sum gained in each period of 33da.; $\$276.52 \times 21 = \$5806.92$ = the whole amount gained on 30 days' notes in 693 days; and $\$50000 \div .9895 = \$50530.57$; $\$50530.57 - \$50000 = \$530.57$; $\$530.57 \times 11 = \$5836.27$ = the whole sum gained on 60 days' notes in 693 days; $\$5836.27 - \$5806.92 = \$29.35$ = the amount which is gained on notes of 60 days *more* than on notes of 30 days, Ans.

15. $450 \times \$3.50 = \$1575$; $450 \times \$4.00 = \$1800$; $\$1800 \div .0305 = \$54.90$; $\$1800 - \$54.90 = \$1745.10$; $\$1745-.10 - \$1575 = \$170.10$, Ans.

16. $6\frac{1}{2} - 6 = \frac{1}{2}$ per cent. $6\frac{1}{2} : \frac{1}{2} : : \$1.00 : \$0.077$, the interest of $\$1$ for the required time at 6 per cent.; $\$0.06 : \$0.077 : : 12$ months : 15m. 12d., Ans.

STOCKS.

(Art. **400**, p. **296**.)

2. $\$24360 \times 1.35 = \$32886$, Ans.

3. $15 \times \$100 \times 1.13 = \$1695$, Ans.

4. $10 \times \$100 \times .85 = \$850$, Ans.

5. $30 \times \$100 = \$3000$; $\$3000 \times 1.08\frac{3}{4} = \$3262.50$, amount; $\$3262.50 - \$3000 = \$262.50$, premium, Ans.

6. $25 \times \$50 \times .97 = \$1212.50$, Ans.

7. $22 \times \$250 = \$5500$; $\$5500 \times .95 = \$5225$, amount paid; $\$5500 - \$5225 = \$275$, Ans.

8. $\$50000 \times 1.17 = \$58500$, Ans.

9. $\$19500 \times .93 = \$18135$; $\$19500 \times 1.03 = \$20085$; $\$20085 - \$18135 = \$1950$, Ans.

(Art. **401**, p. **297**.)

2. $\$6210 \div 1.035 = \$6000$, Ans.

3. $\$1155 \div 11 = \$105$; $\$105 \div 1.05 = \$100$, Ans.

4. $\$1230 \div 41 = \$30$; $\$30 \div .60 = \$50$, Ans.

5. $\$2052 \div 1.08 = \$1900$; $\$2052 - \$1900 = \$152$, Ans.

6. $\$16245 \div .95 = \$17100$; $\$17100 - \$16245 = \$855$, Ans.

7. $862.50 \div 57.50 = 15$ shares; $\$862.50 \div 1.15 = \$750$; $\$862.50 - \$750 = \$112.50$, Ans.

8. $\$1000 \times .88 = \$880$; $7920 \div 880 = 9$ bonds; $\$1000 - \$880 = \$120$; $\$120 \times 9 = \$1080$, Ans.

(Art. **402**, p. **298**.)

2. $1.00 - .25 = .75$; $.06 \div .75 = .08$, Ans.

3. $.11 \div 1.10 = 10$ per cent., Ans.

4. (402, note.) $.12 \div .8 = 1.50$; $1.50 - 1.00 = .50$, Ans.

5. $.09 \div 1.25 = .07\frac{1}{5}$; $.06 \div .75 = .08$; $.08 - .07\frac{1}{5} = \frac{4}{5}$ per cent.; therefore, it is the better investment by $\frac{4}{5}$ per cent. to buy the 6 per cent. stock.

6. $.05 \div .07 = .71\frac{3}{7}$; $1.00 - .71\frac{3}{7} = .28\frac{4}{7}$, Ans.

7. $.05 \div .80 = .06\frac{1}{4}$; $\$20000 \times .06\frac{1}{4} = \$1250$; $\$20000 \times .06 = \$1200$; $\$1250 - \$1200 = \$50$, Ans.

BROKERAGE AND COMMISSION.

(Art. **407**, p. **299**.)

2. $\$18768 \times .01\frac{3}{4} = \$328.44$, Ans.

3. $\$112.25 \times 12 = \$1347$; $1347 \times .00\frac{1}{4} = \$3.36\frac{3}{4}$, Ans.

4. $\$12.25 \times 700 = \$8575$; $\$8575 \times .01\frac{3}{4} = \$150.06\frac{1}{4}$, Ans.

5. $50 \div 10000 = .005 = \frac{1}{2}$ per cent., Ans.

6. $173 \times \$8.95 \times .01\frac{7}{8} = \$29.03\frac{5}{32}$, Ans.

7. $37 \times \$107.75 \times .00\frac{3}{8} = \$14.95\frac{1}{32}$, Ans.

8. $395£. 15s. 5d. = 94985d.$; $94985 \times .02\frac{1}{4} = 2137.1625d. = 8£. 18s. 1\frac{65}{100}d.$, Ans.

9. $5.46 \div 364 = .01\frac{1}{2}$, Ans.

(Art. **408**, p. **301**.)

2. $\$1976 \div 1.04 = \$1900$; $\$1976 - \$1900 = \$76$, Ans.

3. $\$8341.50 \div 1.005 = \$8300$; $\$8300 \div 83 = 100$ shares; $\$8341.50 - \$8300 = \$41.50$ brokerage, Ans.

4. $\$8960 \div 1.02 = \$8784.31\frac{18}{51}$; $\$8960 - \$8784.31\frac{18}{51} = \$175.68\frac{33}{51}$, Ans.

14

5. \$5150 ÷ 1.03 = \$5000; \$5150 — \$5000 = \$150, Ans.

6. \$285 ÷ .015 = \$19000, Ans.

7. \$740.83¾ — \$7.50 = \$733.33¾ = the bill minus the cart-
age; \$733.33¾ ÷ 1.015 = \$722.50; \$733.33¾ —
\$722.50 = \$10.83¾ = commission; 250 × 34 = 8500
lb. of· sugar; \$722.50 ÷ 8500 = \$0.08½ per pound,
Ans.

8. 987£. 18s. 6d. × .0225 = 22£. 4s. $6\frac{1}{200}$d.; 987£. 18s.
6d. — 22£. 4s. $6\frac{1}{200}$d. = 965£. 13s. $11\frac{41}{200}$d.; 965£.
13s. $11\frac{41}{200}$d. × 100 = 96569£. 13s. 4½d.; 96569£.13s.
4½d. ÷ 101⅞ = 947£. 18s. $5\frac{44}{815}$d.; 947£. 18s. $5\frac{44}{815}$d.
= 227501$\frac{44}{815}$d.; 1£. 3s. 8d. = 284d.; 227501$\frac{44}{815}$ ÷
284 = 801$\frac{3576}{57965}$yds., Ans.

ACCOUNT OF SALES.

(ART. 410, p. 303.)

(2.)

Amount of sales,	. . . .	.	\$2671.40
Freight,	. . . . .	\$185.50	
Commission at 3 per cent. on \$2671.40,		74.57	
Cartage, cooperage, &c.,	. .	26.00	
Storage and insurance, .	. .	63.24	
	Amount of charges,	. . .	\$349.31
Net proceeds to J. B. & Co.,	. . .	.	\$2322.09

PROFIT AND LOSS.

(ART. 412, p. 304.)

2. \$5 × .80 = \$4, Ans.

3. \$8.50 × 1.10 = \$9.35, Ans.

4. $0.42 $\times$ .95 = $0.399, Ans.

5. $2500 $\times$ 1.20 = $3000, Ans.

(ART. **413**, p. **304**.)

2. $4 $\div$ .80 = $5, Ans.

3. $96 $\div$ 1.20 = $80; $80 $\div$ 10 = $8, Ans.

4. $12.50 $\div$ 1.17 = 10.683\frac{89}{117}$, Ans

5. $6.125 $\div$ .875 = $7.00, Ans.

(ART. **414**, p. **305**.)

2. $7 — $6.125 = $0.875; .875 $\div$ 7 = 12$\frac{1}{2}$ per cent., Ans.

3. $225 — $200 = $25; 25 $\div$ 200 = 12$\frac{1}{2}$ per cent., Ans.

4. 500 $\div$ 2500 = 20 per cent., Ans.

5. $64.86,4$\frac{32}{37}$ was given for the cloth, and $2.50 $\times$ 24 = $60.00 was received for it; therefore the loss was $64.86,4$\frac{32}{37}$ — $60.00 = $4.86,4$\frac{32}{37}$, or $\dfrac{4864\frac{32}{37}}{64864\frac{32}{37}}$ of its value, = .075 = 7$\frac{1}{2}$ per cent., Ans.

(ART. **415**, p. **306**.)

2. $7 $\div$ 1.75 = $4 prime cost; $4 — $3 = $1, loss on 1yd.; 1 $\div$ 4 = .25 loss, Ans.

3. $1.25 $\div$ .85 = 1.470\frac{10}{17}$ par value of corn; 1.647\frac{1}{17}$ — 1.470$\frac{10}{17}$ = .176$\frac{8}{17}$ gain per bushel if sold for 1.647\frac{1}{17}$; .176$\frac{8}{17}$ $\div$ 1.470$\frac{10}{17}$ = .12, Ans.

4. 1.647\frac{1}{17}$ $\div$ 1.12 = 1.470\frac{10}{17}$; 1.470\frac{10}{17}$ — $1.25 = 0.220\frac{10}{17}$ loss per bushel if sold at $1.25; .220$\frac{10}{17}$ $\div$ 1.470$\frac{10}{17}$ = 15 per cent., Ans.

5. $75 $\div$ .75 = $100 cost of the horse; $100 $\times$ 1.30 = $130 real value of horse; $130 — $75 = 55, Ans.

6. $1.25 $\div$ .75 = 1.66\frac{2}{3}$ cost per pound; 1.66\frac{2}{3}$ — $1.40 = .26$\frac{2}{3}$ loss per lb. if sold at $1.40; $.26$\frac{2}{3}$ $\div$ 1.66$\frac{2}{3}$ = 16 per cent. loss, Ans.

MISCELLANEOUS EXAMPLES.

(Page **306.**)

1. $100 × 1.15 = $115; $115 − $100 = $15 gain on 1 share; $120 ÷ 15 = 8 shares, Ans.

2. $250 × 1.09 = $272.50 cost per share; $272.50 − $25 = $247.50 = what one share was sold for; $250 − $247.50 = $2.50 loss by each share on the par value; 2.50 ÷ 250 = .01 discount, Ans.

3. $19200 ÷ .96 = $20000; $20000 × .07 = $1400 = income from 7 per cent. stocks; $19200 ÷ .80 = $24-000; $24000 × .05 = $1200 = income from 5 per cent. stocks; $1400 − $1200 = $200 greater income from 7 per cent. stocks, Ans.

4. If each share of $250 should pay 10 per cent., the dividend would be $25; then, by proportion, $25 : $15 :: $250 : $150, Ans.

5. $1.00 − $0.20 = $0.80; $0.80 × .015 = .012 commission on each bill of $1; $0.80 + $0.012 = $0.812 = the cost per cent. of the bills; .812 × $200 = $162.40 the amount lost on the worthless bills; $162.40 + $364 = $526.40 = sum which must accrue from the bills sold at par to make a net gain of $364; $526.40 ÷ .188 (the gain on each bill of $1 sold at par) = $2800 = amount of bills sold at par; $2800 + $200 (amount of the worthless bills) = $3000, Ans.

6. $5640 − $76.50 = $5563.50 = proceeds of cotton *minus* the storage; $5563.50 − $5422.50 (sum remitted) = $141 commission; $141 ÷ 5640 = .02½ the per cent. of commission, Ans.

7. The horse was bought for 70 per cent of his real worth; and .70 × .75 = .525; that is, the horse was sold for 52½ per cent. of his original worth, which is a loss of 1.00 − .525 = 47½ per cent.; but by the question 47½ per cent.

of the original value = \$55; therefore $47\frac{1}{2}$: \$55 : :
$52\frac{1}{2}$: \$60.78$\frac{18}{15}$, the sum received for the horse, Ans.

8. \$35 ÷ 1.04 = \$33.65,3$\frac{11}{13}$ present worth of \$35; \$33-
65,3$\frac{11}{13}$ — \$30 = \$3.65,3$\frac{11}{13}$ gain, Ans.

9. 63gal. — 15gal. = 48gal.; \$2.21,6$\frac{2}{3}$ × 48 = \$106.40,
price sold at; \$112 — \$106.40 = \$5.60 loss; \$112 :
\$5.60 : : 100 : 5 per cent. loss, Ans.

10. 63gal. — 15gal. = 48gal. left: \$2.21,6$\frac{2}{3}$ × 48 = \$106-
.40; \$106.40 ÷ .95 = \$112, Ans.

11. \$9.00 × .90 = \$8.10, what 1bbl. of the damaged flour is
sold for; \$8.10 × 25 = \$202.50, proceeds from 25bbl.
of damaged flour; \$9.50 × 25 = \$237.50; \$202.50 +
\$237.50 = \$440, the avails of the flour; \$9 × 50 =
\$450, cost of the flour; \$450 — \$440 = \$10 loss, Ans.

12. \$6135 ÷ 1.02$\frac{1}{4}$ = \$6000 = sum paid for the flour;
\$6000 × 1.20 = \$7200; \$7200 — \$6135 = \$1065;
\$1065 — \$31.63 (storage, &c.) = \$1033.37, Ans.

13. \$102.50 × 90 = \$9225 ÷ 2$\frac{1}{2}$•= \$3690, the sum remit-
ted; \$3690 — \$90 = \$3600 ÷ 95 = 37T. 17cwt. 3qr.
14$\frac{9}{19}$lb., Ans.

PARTNERSHIP.

(Art. 417, p. 309.)

2. \$4000 ÷ 10000 = 40 per cent. gain; \$3000 × 40 =
\$1200, Jones' part.
 \$2000 × .40 = \$800, Weston's part.
 \$5000 × .40 = \$2000, Sprague's part.

3. \$18780 : \$ 6780 : : \$2000 : \$ 722.044, C's part.
 \$18780 : \$12000 : : \$2000 : \$1277.956, D's part.

4. Harvey   \$2500 × .05 = \$125, Harvey's loss.
 Blake   \$3000 × .05 = \$150, Blake's loss.
 Horsford \$5500 × .05 = \$275, Horsford's loss.

5. $ 332.50 — $ 120 = $ 212.50 to be divided between Elliott and Mahew; $ 850 : $ 212.50 : : $ 500 : $ 125, Elliott's share; $ 850 : $ 212.50 : : $ 350 : $ 87.50, Mahew's share, Griswold's gain will bear the same relation to his stock that Mahew's or Elliott's does; therefore $ 125 : $ 500 : : $ 120 : $ 480, the value of Griswold's stock.

6. $ 5000 + $ 6500 + $ 7500 = $ 19000, amount of stock; $ 19000 × .40 = $ 7600; $ 7600 × .90 = $ 6840, net gain.

$$\left.\begin{array}{l} \$ 19000 \;:\; \$ 6840 \;::\; \$ 5000 \;:\; \$ 1800,\ \text{A's gain,} \\ \$ 19000 \;:\; \$ 6840 \;::\; \$ 6500 \;:\; \$ 2340,\ \text{B's gain,} \\ \$ 19000 \;:\; \$ 6840 \;::\; \$ 7500 \;:\; \$ 2700,\ \text{C's gain,} \end{array}\right\} \text{Ans.}$$

7. $ 40000 capital; $ 2000 + $ 4500 + $ 2500 + $ 1500 = $ 10500 = whole gain.

$$\left.\begin{array}{l} \$ 10500 \;:\; \$ 40000 \;::\; \$ 2000 \;:\; \$\;\; 7619.04\tfrac{16}{21}\ \text{A's,} \\ \$ 10500 \;:\; \$ 40000 \;::\; \$ 4500 \;:\; \$ 17142.85\tfrac{15}{21}\ \text{B's,} \\ \$ 10500 \;:\; \$ 40000 \;::\; \$ 2500 \;:\; \$\;\; 9523.80\tfrac{20}{21}\ \text{C's,} \\ \$ 10500 \;:\; \$ 40000 \;::\; \$ 1500 \;:\; \$\;\; 5714.28\tfrac{12}{21}\ \text{D's,} \end{array}\right\} \text{Ans.}$$

8. $ 7500 gains $ 1000 + $ 500 = $ 1500, which is 20 per cent. on the principal; hence,

$$\left.\begin{array}{l} \$ 2000 \times .20 = \$\; 400,\ \text{A's gain,} \\ \$ 3000 \times .20 = \$\; 600,\ \text{B's gain,} \\ \$ 400 + \$ 600 + \$ 500 = \$ 1500,\ \text{C's gain,} \end{array}\right\} \text{Ans.}$$

9. Let A = 1.00; then B = 1.50; C = 1.25; $ 30000 × .25 = $ 7500 whole profits; A + B + C = 3.75.

$$\left.\begin{array}{l} 3.75 \;:\; \$ 7500 \;::\; 1.00 \;:\; \$ 2000,\ \text{A's gain,} \\ 3.75 \;:\; \$ 7500 \;::\; 1.50 \;:\; \$ 3000,\ \text{B's gain,} \\ 3.75 \;:\; \$ 7500 \;::\; 1.25 \;:\; \$ 2500,\ \text{C's gain,} \end{array}\right\} \text{Ans.}$$

10. The fractions $\tfrac{1}{3}, \tfrac{1}{4}, \tfrac{1}{5}$, are as the fractions $\tfrac{20}{60}, \tfrac{15}{60}, \tfrac{12}{60}$, or as 20, 15, 12; 20 + 15 + 12 = 47; $\tfrac{20}{47}$ = Walker's, $\tfrac{15}{47}$ = Edwards', $\tfrac{12}{47}$ = Armstrong's share of the gain; but Armstrong's is divided between Walker and Edwards, and Walker's share is to Edwards' as 20 to 15; hence

Walker's share is $\frac{2}{4}\frac{9}{7} + \frac{3}{3}\frac{9}{5}$ of $\frac{1}{4}\frac{2}{7} = \frac{4}{7}$; and Edwards'
share is $\frac{1}{4}\frac{5}{7} + \frac{1}{3}\frac{5}{5}$ of $\frac{1}{4}\frac{2}{7} = \frac{3}{7}$; then

$$\frac{4}{7} \text{ of } \$50000 = \$28571.42\frac{4}{7} \text{ Walker's,}$$
$$\frac{3}{7} \text{ of } \$50000 = \$21428.57\frac{4}{7} \text{ Edwards',}$$ Ans.

(ART. 418, p. 311.)

2.   BY ANALYSIS.

$3200 for 12 months is the same as $38400 for 1 month;
and $4200 for 8 months is the same as $33600 for 1
month. Therefore the whole stock may be considered
$38400 + $33600 = $72000. Goodwin's share of the
profit will therefore be $\frac{38400}{72000} = \frac{8}{15}$ of $240 = $128;
Blunt's share will be $\frac{33600}{72000} = \frac{7}{15}$ of $240 = $112, Ans.

FORM OF STATEMENT.

$3200 × 12 = $38400, G's product.
$4200 × 8 = $33600, B's product.
            ———
            $72000

$72000 : $240 : : $38400 : $128, G's gain,
$72000 : $240 : : $33600 : $112, B's gain, Ans.

3.   BY ANALYSIS.

24 oxen for 8 weeks is 1 ox for 192 weeks; 18 oxen for 12
weeks is 216 oxen for 1 week; and 12 oxen for 10 weeks
is 1 ox for 120 weeks. The amount of pasturing is 192 +
216 + 120 = 528 weeks. A's share is $\frac{192}{528} = \frac{4}{11}$ of
$26.40 = $9.60; B's share is $\frac{216}{528} = \frac{9}{22}$ of $26.40 =
$10.80; C's share is $\frac{120}{528} = \frac{5}{22}$ of $26.40 = $6.00, Ans.

FORM OF STATEMENT.

24 oxen × 8 = 192 oxen.
18 oxen × 12 = 216 oxen.
12 oxen × 10 = 120 oxen.
              ———
              528 oxen.

528 : $26.40 : : 192 : $ 9.60 what A pays,
528 : $26.40 : : 216 : $10.80 what B pays, Ans.
528 : $26.40 : : 120 : $ 6.00 what C pays,

(4.)

$$\$300 \times 5 = \$1500$$
$$\$400 \times 8 = \$3200$$
$$\$500 \times 3 = \underline{\$1500}$$
$$\$6200$$

$\$6200 : \$100 :: \$1500 : \$24.19\frac{11}{31}$ Barclay's, ⎫
$\$6200 : \$100 :: \$3200 : \$51.61\frac{9}{31}$ Hickman's, ⎬ Ans.
$\$6200 : \$100 :: \$1500 : \$24.19\frac{11}{31}$ Oliver's, ⎭

5. $\$21 + \$40 + \$24 = \$85$, whole gain; and

$\frac{21}{85} =$ A's for 7mo., and $\frac{21}{85} \div 7 = \frac{3}{85}$, A's for 1mo.

$\frac{40}{85} =$ B's for 8mo., and $\frac{40}{85} \div 8 = \frac{5}{85}$, B's for 1mo.

$\frac{24}{85} =$ C's for 12mo., and $\frac{24}{85} \div 12 = \frac{2}{85}$, C's for 1mo.

$\frac{3}{85} + \frac{5}{85} + \frac{2}{85} = \frac{10}{85}$; hence

$\frac{10}{85} : \frac{3}{85} :: \$1000 : \$300 =$ A's capital, ⎫
$\frac{10}{85} : \frac{5}{85} :: \$1000 : \$500 =$ B's capital, ⎬ Ans.
$\frac{10}{85} : \frac{2}{85} :: \$1000 : \$200 =$ C's capital, ⎭

6. Let $100 =$ gain of each; $100 + 100 = 200$, gain of both.

$\frac{100}{288} =$ White's gain in 12mo., and $\frac{100}{288} \div 12 = \frac{2}{48}$, White's gain in 1mo.

$\frac{100}{288} =$ D's gain in 8mo., and $\frac{100}{288} \div 8 = \frac{3}{48}$, D's gain in 1mo.

$\frac{2}{48} + \frac{3}{48} = \frac{5}{48}$, gain of both in 1mo., or in any equal time.

$\frac{5}{48} : \frac{2}{48} :: \$6300 : \$2520$, White's capital, ⎫
$\frac{5}{48} : \frac{3}{48} :: \$6300 : \$3780$, Daniel's capital, ⎬ Ans.

(7.)

$\$4000 \times 4 = 16000$	$\$3000 \times 10 = 30000$
$\underline{\ \ 500}$	$\underline{1500}$
$\overline{4500} \times 12 = 54000$	$\overline{1500} \times 4 = \ \ 6000$
$\underline{1000}$	$\underline{3000}$
$\overline{3500} \times 4 = \underline{14000}$	$\overline{4500} \times 6 = \underline{27000}$
A's product $= \overline{84000}$	B's product $= \overline{63000}$

$2000 × 6 = 12000
2000
———————
4000 × 8 = 32000
2000
———————
6000 × 2 = 12000
1500
———————
4500 × 4 = 18000
———————
C's product = 74000

A's product, 84000
B's product, 63000
C's product, 74000
———————
221000

$221000 : $4420 :: $84000 : $1680, A's gain, ⎫
$221000 : $4420 :: $63000 : $1260, B's gain, ⎬ Ans.
$221000 : $4420 :: $74000 : $1480, C's gain, ⎭

(8.)

$12000 × 6 = 72000
2500
———————
9500 × 3 = 28500
1000
———————
8500 × 3 = 25500
———————
G's product = 126000

$18000 × 6 = 108000
2500
———————
15500 × 3 = 46500
6000
———————
9500 × 3 = 28500
———————
T's product = 183000

126000 + 183000 = 309000, sum of products.

309000 : 126000 :: $15000 : 6116\frac{52}{103}$, G's sha., ⎫
309000 : 183000 :: $15000 : 8883\frac{51}{103}$, T's sha., ⎬ Ans.

(9.)

$4000 × 6 = 24000
2000
———————
2000 × 18 = 36000
———————
J's product = 60000

$3500 × 8 = 28000
1500
———————
2000 × 16 = 32000
———————
E's product = 60000

$2500 × 10 = 25000
2000
———————
4500 × 14 = 63000
———————
B's product = 88000

J's product 60000
E's   "   60000
B's   "   88000
———————
208000

208000 : 60000 :: $1041.80 : 300.51\frac{12}{13}$, J's, ⎫
208000 : 60000 :: $1041.80 : 300.51\frac{12}{13}$, E's, ⎬ Ans.
208000 : 88000 :: $1041.80 : 440.76\frac{2}{13}$, B's, ⎭

10. This problem may be solved by analysis, thus : $10000 gain
    $22.50 in 12mo., and gain $\frac{22500}{12}$ = $187.50 in 1mo., and
    $\frac{187.50}{10000}$ = .01875, the gain of $1 in 1mo.; and $2800 ×
    .01875 = $52.50 = Parkman's gain in 1mo.; and
    420 ÷ 52.50 = 8mo., the time Parkman's money was
    in trade; and $3600 × .01875 = $67.50, Delano's gain
    in 1mo.; hence, 405 ÷ 67.50 = 6, the number of
    months Delano's money was in trade. To get the rate per
    cent., divide the gain for 1y. by the capital, 2250 ÷
    10000 = .22½ per cent., Ans.

The problem may be solved by Art. 360. Obtaining the rate
    per cent. as before, divide the gain by the interest of the
    principal for one year.

$2800 × .22½ = $630; 420 ÷ 630 = $\frac{2}{3}$y. = 8mo., P's
    time.

$3600 × .22½ = $810; 405 ÷ 810 = ½y. = 6mo., D's
    time.

BANKRUPTCY.

(Art. 420, p. 313.)

2. $52384 — $1584 = $50800; 50800 ÷ 63500 = .80;
   $8361.55 × .80 = $6689.24, amount received by Day-
   ton, Ans.

3. $600 + $760 + $840 + $800 = $3000; $2275 ÷ 3000
   = $0.75⅚, the amount paid on $1;

   $600 × .75⅚ = $455.00,   A's dividend, ⎫
   $760 × .75⅚ = $576.33⅓, B's   "   ⎬ Ans.
   $840 × .75⅚ = $637.00,   C's   "   ⎪
   $800 × .75⅚ = $606.66⅔, D's   "   ⎭

4. $400 + $300 + $1000 = $1700; 600 ÷ 1700 = .35$\frac{5}{17}$;

   $ 400 × .35$\frac{5}{17}$ = 141.17\frac{11}{17}$, A's share, ⎫
   $ 300 × .35$\frac{5}{17}$ = 105.88\frac{4}{17}$, B's   "   ⎬ Ans.
   $1000 × .35$\frac{5}{17}$ = 352.94\frac{2}{17}$, C's   "   ⎭

5. $180000+$40000+$12875 = $232875, and this amount contains both the sum distributed to the creditors and the $3\frac{1}{2}$ per cent. charges; therefore, $232875 ÷ 1.03$\frac{1}{2}$ = $225000 = the amount distributed to the creditors; and 225000 ÷ 300000 = .75, the per cent. paid; $1360.60 × .75 = $1020.45, amount received by Henderson, Ans.

TAXES.

(Art. **425**, p. **316**.)

2. $1310 × .005 = $ 6.55, amount on F's property.
   $1.50 × 7 = $10.50, Forster's poll tax.
             $17.05, Forster's tax, Ans.

3. $415.35 × .005 = $2.07675, Ans.

4. $3506300 = ———, taxable property;
  $14018.90 — $3500 = $10518.90, to be assessed on property;
  $10518.90 ÷ 3506300 = .003 = tax on $1;
  $29010 × .003 = $87.03 = A's property tax;
  $87.03 + $2 = $89.03 = amount of A's tax, Ans.

(Art. **426**, p. **316**.)

2. $10450 ÷ .95 = $11000, Ans.

3. $9700 ÷ .97 = $10000 = assessment; $10000 — $9700 = $300 = cost of collection, Ans.

4. $756000 = ———, taxable property;
  $18000 ÷ .97 = $18556.70 = amount of tax to be raised;
  $450 × 1.50 = $675 = amount assessed on the polls;
  $18556.70 — $675 = $17881.70 = amount to be assessed on the property;
  $17881.70 ÷ 756000 = $0.0236 (nearly), tax on each dollar.

$15600 × .0236 = $368.16, A's property tax;

$1.50 × 3 = $4.50, A's poll tax;

$368.16 + $4.50 = $372.66, amount of A's tax, Ans.

(Art. 427, p. 317.)

2. $500 + $150 + $30 = $680, amount of expenditures;

$680 — $350 = $330, amount of rate money;

$330 ÷ 5500 = $0.06, rate per day;

60 + 60 + 30 = 150 days' attendance for A;

150 × $0.06 = $9, A's bill, Ans.

3. $150 + $18.50 = $168.50, amount of expenditures;

$168.50 — $63.50 = $105, amount of rate money;

$105 ÷ 3000 = $0.035, rate per day for each pupil;

121 × $0.035 = $4.235, A's rate bill, $\Big\}$ Ans.

173 × $0.035 = $6.055, B's rate bill,

GENERAL AVERAGE.

(Art. 432, p. 320.)

(2.)

CONTRIBUTORY INTERESTS.		LOSS FOR GENERAL BENEFIT.	
Vessel,	$31500	Thrown overboard,	$6500
Cargo,	6235	Repairs less ⅓,	700
Freight less ½,	2080	Cost of detention,	142
Entire contrib. int.,	$39815	Entire loss,	$7342

$7342 ÷ 39815 = .18½ (nearly), the loss per cent.;

$31500 × .18½ = $5827.50, am't payable by vessel;

2080 × .18½ = 384.80, am't payable by freight;

2145 × .18½ = 396.825, am't pay'e by Manning & Bro.;

1460 × .18½ = 270.10, am't pay'e by Anderson & Fiske;

960 × .18½ = 177.60, am't payable by Smidt & Huber;

670 × .18½ = 123.95, am't pay'e by Greenwood & Co.;

1000 × .18½ = 185.00, am't payable by A., R., & H.

EQUATION OF PAYMENTS.

(Art. **435**, p. **322**.)

3. $200 to be paid in 3 months is the same as $100 in 6 months; and $300 in 5 months is the same as $100 in 15 months; and $500 in 10 months is the same as $100 in 50 months. If, then, $100 is to be paid in $6 + 15 + 50 = 71$ months, $1000 should be paid in $\frac{1}{10}$ of 71 months $= \frac{71}{10} = 7$mo. 3da., Ans.

OPERATION.

$$
\begin{array}{r}
\$\ 200 \times 3 = 600 \\
\$\ 300 \times 5 = 1500 \\
\$\ 500 \times 10 = 5000 \\
\hline
\$1000 \qquad 1000)7100(7\tfrac{1}{10} \text{ months, Ans.} \\
7000 \\
\hline
100
\end{array}
$$

4. $2000 ×

$$
\begin{array}{r}
500 \times 12 = 6000 \\
4000 \times 24 = 96000 \\
\hline
6500 \qquad 102000
\end{array}
$$

$102000 \div 6500 = 15\frac{9}{13}$mo. $= 15$mo. $20\frac{10}{13}$da., Ans.

5. $50 to be paid in 2 months is the same as $100 in 1 month; $150 in 8 months is the same as $100 in 12 months. The equated time, then, will be as if $100 were to be paid in 1 month, $100 in 5 months, and $100 in 12 months; that is, $100 in $1 + 5 + 12 = 18$ months; consequently $300 in $\frac{1}{3}$ of 18 months $= 6$ months, Ans.

OPERATION.

$$
\begin{array}{r}
\$\ 50 \times 2 = 100 \\
\$100 \times 5 = 500 \\
\$150 \times 8 = 1200 \\
\hline
\$300 \qquad 300)1800(6 \text{ months, Ans.} \\
1800
\end{array}
$$

15

6. The first bill due is that for $600, on May 1.

> Due May   1, $600
> "   July   7,   370 ×   67 =   24790
> "   July 15,   560 ×   75 =   42000
> "   Oct. 20,   420 × 172 =   72240
>   1950·   139030

139030 ÷ 1950 = 71 days, nearly; May 1 + 71 days = July 11, Ans.

7. The first bill due is that for $675.25, on June 1, 1855.

> Due June   1, $675
> "   Nov.   4,   376 × 156 =   58656
> '   "   25,   822 × 177 =   145494
>   1856.
> Due June   1,   961 × 366 =   351726
> "   Apr.   1,   145 × 305 =   44225
> "   Aug. 10,   811 × 436 =   353596
> "   "   12,   568 × 438 =   248784
> "   "   15,   370 × 441 =   163170
>   4728   1365651

1365651 ÷ 4728 = 289 days; June 1, 1855 + 289 days = March 16, 1856, Ans.

(ART. 436, p. 324.)

2. As ½ of $144 is paid 7 months before it is due, it is evident the remainder, $72, is payable in 14 months; but $48 of this sum is paid in 4 months, which is 10 months before it is due. There will then remain $24 unpaid; and the question will be, How long may $24 be kept to pay the interest of $72 for 10 months? $72 for 10 months is the same as $1 for 720 months; and $1 for 720 months is the same as $24 for $\frac{1}{24}$ of 720 months = 720 ÷ 24 = 30 months. To the 30 months we add the 4 months, and we have the whole equated time; thus, 30 + 4 = 34 months = 2y. 10mo., Ans.

OPERATION.

$$144 \times 7 = 1008$$
$$\overline{72}$$
$$\underline{48} \times 4 = \underline{192}$$
$$\overline{120} \qquad 24)\overline{816}(34\text{mo.} = 2\text{y. }10\text{mo., Ans.}$$
$$\overline{24} \qquad \quad \underline{72}$$
$$\qquad\qquad\quad \overline{96}$$
$$\qquad\qquad\quad \underline{96}$$

3. The payments in this problem were not due before Nov. 23, 1856; the days are reckoned from the time each payment was made to the end of 8 months, Nov. 23.

$$\$\,8000$$

Paid April	5,	1200	× 232 =	278400
" July	4,	1500	× 142 =	213000
" Sept.	25,	1800	× 59 =	106200
" Oct.	1,	1000	× 53 =	53000
" Nov.	20,	500	× 3 =	1500
		6000		652100

Balance unpaid, $\$\,2000$

$652100 \div 2000 = 326$ days; Nov. 23, 1856 $+$ 326 days $=$
[Oct. 15, 1857, Ans.

4. $\$\,133\frac{1}{3}$ in 2 months is the same as $\$1$ in $266\frac{2}{3}$ months; and $\$266\frac{2}{3}$ in 3 months is the same as $\$1$ in 800 months; $\$400$ in 6 months is the same as $\$1$ in 2400 months; and $\$1$ in $266\frac{2}{3} + 800 + 2400 = 3466\frac{2}{3}$ months is the same as $\$800$ for $\frac{1}{800}$ of $3466\frac{2}{3}$ months $=$ $3466\frac{2}{3} \div 800 = 4\frac{1}{3}$ months. And if $\frac{1}{2}$ of $\$800$ be paid down, the remainder, $\$400$, may be kept twice $4\frac{1}{3}$ months $= 8\frac{2}{3}$ months, Ans.

OPERATION.

$$\$133\tfrac{1}{3} \times 2 = 266\tfrac{2}{3}$$
$$266\tfrac{2}{3} \times 3 = 800$$

$$\begin{array}{r} 400 \times 6 = 2400 \\ \hline 800 \qquad\qquad 400)3466\tfrac{2}{3}(8 \text{ months.} \\ 400 \qquad\qquad 3200 \\ \hline 400 \qquad\qquad 266\tfrac{2}{3} \\ 30 \end{array}$$

$$400)8000(20 \text{ days.}$$
$$\underline{800}$$
$$0$$

5. The payments made in this bill were not due before March 25, 1857, 6 months from Sept. 25, 1856.

1856.   3051

    Paid Oct.   4, $476 \times 172 = 81872$

     " Nov. 12, $375 \times 133 = 49875$

     " Dec.   5, $800 \times 110 = 88000$

1857.

     " Jan.   1, $200 \times 83 = 16600$

                1851   236347

Balance unpaid, $\$1200$

$236347 \div 1200 = 197$ days, nearly; March 25, 1857 + 197 [days = Oct. 8, 1857, **Ans.**

AVERAGING OF ACCOUNTS.

(Art. 438, p. 327.)

2. The first item in this account was due July 4, 1855.

Days reckoned from July 4, 1855.	Days reckoned from July 4, 1855.
July 4, $876	Aug. 10, $816 \times 37 = 11692$
Aug. 20, $816 \times 47 = 38352$	Sept. 1, $675 \times 59 = 39825$
Aug. 29, $178 \times 56 = 9968$	Sept. 25, $512 \times 83 = 42496$
Sept. 25, $387 \times 83 = 32121$	Nov. 20, $162 \times 189 = 22518$
Dec. 5, $419 \times 154 = 64526$	Dec. 1, $100 \times 150 = 15000$
$2176 144967	$1765 131531

$2176 — $1765 = $411, difference of items; 144967 — 131531
= 13436, difference of products; 13436 ÷ 411 = 32 +
days; July 4, 1855 + 33 days = Aug. 6, 1855, Ans.

3. The first item due in this account is the cash payment of
$700, April 1; from this date the time is reckoned.

Due 1856.		Due 1856.	
Sept. 1,	$721 × 153 = 110313	April 1,	$700
Sept. 20,	815 × 172 = 140180	Nov. 30,	570 × 243 = 138510
Oct. 11,	588 × 193 = 113484	July 20,	500 × 110 = 55000
Oct. 30,	300 × 212 = 63600	Sept. 25,	100 × 177 = 17700
Dec. 15,	625 × 258 = 161250	1857.	
1857.		Mar. 30,	750 × 363 = 272250
Jan. 18,	560 × 292 = 163520	April 30,	330 × 394 = 130020
*Feb. 28,	685 × 333 = 228105	May 20,	500 × 414 = 207000
Mar. 25,	365 × 358 = 130670		$3450 820480
	$4659 1111122		

1111122 — 820480 = 290642, balance of products;

$4659 — $3450 = $1209, balance of the items;

290642 ÷ 1209 = 240 days; 240 days forward from April 1,
1856 = November 27, 1856, Ans.

* See note 2, p. 350, National Arithmetic.

4. In this problem the time is reckoned from the first date in
the account to the date of each item, it being due from
date.

1856.	(Time from March 1.)	1856.	(Time from April 1.)
Mar. 1,	$86.25	April 1,	$48.25
April 7,	18.15 × 37 = 671.55	May 20,	90.10 × 49 = 4414.90
June 15,	48.26 × 106 = 5115.56	June 17,	12.50 × 77 = 962.50
July 21,	91.20 × 142 = 12950.40	July 4,	20.00 × 94 = 1880.00
Aug. 1,	30.00 × 153 = 4590.00	July 10,	25.00 × 100 = 2500.00
	$223.86 23327.51		$195.85 9757.40

23327.51 ÷ 223.86 = 104 days; March 1 + 104 days =
June 13;

9757.40 ÷ 195.85 = 50 days, nearly; April 1 + 50 days =
May 21;

15*

June 13 is the average time of the debit items becoming due; and May 21 is the average time of the credit items becoming due. From May 21 to June 13 = 23 days = difference between the average dates; then the amount of the smaller side, 195.85 × 23 = 4504.55; 4504.55 ÷ 28.01 (= 223.86 — 195.85, balance of the account) = 161 days nearly; hence, June 13 (the average date of the larger size) + 161 days = Nov. 21, 1856, Ans.

$223.86 — $195.85 = $28.01, balance due Nov. 21; $28.01 ÷ 1.01 (the amount of $1 from Sept. 21, 1856, to Nov. 21, 1856) = $27.73, the cash value of balance, Sept. 1, 1856, Ans.

5. In the adjustment of this account, the time is reckoned for the debit items from the first date; after ascertaining the average time of payment, the 8 months of credit are added.

1856.				1857.		
May 1,	$300			Jan. 1,	$500	
July 7,	760 ×	67 =	50920	*Oct. 18,	482 × 290 =	189780
Sept. 11,	417 ×	133 =	55461	Mar. 19,	750 × 77 =	57750
Nov. 25,	288 ×	208 =	59904	April 1,	210 × 90 =	18900
Dec. 20,	571 ×	233 =	133043	May 25,	100 × 144 =	14400
	$2336		299328		$2042	230830

299328 ÷ 2336 = 128 days; May 1 + 128 days = Sept. 6, 1856.

230830 ÷ 2042 = 113 days; Jan. 1 + 113 days = April 24, 1857.

Sept. 6, 1856, is the average date of the debits; and Sept. 6, 1856 + 8 mo. = May 6, 1857, the average time of the debits becoming due; April 24 is the average time of the credits becoming due. The difference between April 24 and May 6 is 12 days; 2042 (amount of smaller side) × 12 = 24504; 24504 ÷ 294 (balance of the account)

* Due Oct. 18.

= 83 days; May 6 (average time of the larger side) +
83 days = July 28, 1857, Ans.

Also, what will be the value of the balance on Nov. 28, 1857?
$294 is the balance; from July 28 to Nov. 28 are 4
months; the amount of $294 for 4 months = $294 ×
1.02 = $299.88, Ans.

6. In this problem the first item of debits becoming due is that
of $560, on June 1; and the first item of credits becom-
ing due is that of $100, cash paid June 20.

Due June 1, $560.			Due June 20, $100.		
846 × 71 =	60066		820 × 119 =	97580	
728 × 111 =	80808		350 × 79 =	27650	
400 × 59 =	23600		800 × 56 =	44800	
560 × 345 =	193200		630 × 271 =	170730	
600 × 200 =	120000		400 × 130 =	52000	
500 × 343 =	171500		750 × 254 =	190500	
$4194	649174		$3850	583260	

649174 ÷ 4194 = 155 days; June 1 + 155 days = Nov. 3,
1856.

583260 ÷ 3850 = 151 days; June 20 + 151 days = Nov. 18,
1856.

Nov. 18 — Nov. 3 = 15 days, the difference between the ave-
rage date of the debits and credits becoming due; 3850
(the amount of the smaller side) × 15 = 57750; 57750
÷ 344 (the balance of the account) = 168 days. Since
the amount of the larger side becomes due first, Nov. 3,
the 168 days must be counted backward from that date;
therefore, Nov. 3, 1856 — 168 days = May 19, 1856,
Ans.

The time from this date to the time of settlement, 12mo. 5da.;
therefore, $344 × 1.060⅚ = $364.93, the value of the
balance, May 24, 1857, Ans.

(Art. **439**, p. **331**.)

ALFRED HICKS *in account with* KEEN & LEE.

1857.	1857.
Due July 1, $156 × 153 = 23868	Due Feb. 1, $120 × 303 = 36360
" Feb. 3, 100 × 301 = 30100	" July 20, 420 × 134 = 56280
" July 20, 317 × 134 = 42478	" Nov. 1, 300 × 30 = 9000
" " 30, 162 × 124 = 20088	" " 1, 50 × 30 = 1500
" May 15, 100 × 200 = 20000	1858.
1858.	" Feb. 20, 213 × 81 = 17253*
" Jan. 10, 100 × 40 = 4000*	$890 6)120393
$835 6)140534	100 20.065
218 23.422	$990
$1048	

$1048 — $990 = $58 = balance of items.

$23.422 — $20.065 = $3.36 = balance of interest.

$58 + $3.36 = $61.36, balance due Dec. 1, 1857.

* These products becoming due after settlement, are transferred each to its opposite side ; but the items themselves must be added in their own places.

4. In finding the balance of this account, the time is reckoned from the date of each item, since it is due from date.

$200 × 264 = 52800	$300 × 248 = 74400
300 × 198 = 59400	350 × 222 = 77700
250 × 181 = 45250	400 × 205 = 82000
600 × 175 = 105000	320 × 144 = 46080
400 × 125 = 50000	600 × 109 = 65400
500 × 103 = 51500	100 × 95 = 9500
100 × 69 = 6900	200 × 52 = 10400
120 × 14 = 1680	150 × 25 = 3750
$2470 372530	$2420 369230

372530 — 369230 = 3300; 3300 ÷ 6000 = $0.55, bal-
ance of interest at 6 per cent.; $0.55 ÷ 6 = $0.09⅙;
$0.09⅙ × 7 = $0.64, balance of interest at 7 per cent.;

Items of debit, $2470   Balance of items, $50
Items of credit, 2420   Balance of interest, 0.64
Balance of items, $50;   Bal. due Mar. 25, 1857, $50.64, Ans.

ACCOUNTS OF STORAGE.

(ART. 441, p. 333.)

2. 1857.   chests.   da.   prod.
    May 16, Rec.   4560 × 14 = 63840
    " 30, Deliv.   564
            Bal.   3996 × 2 =   7992
    June 1, Deliv. 904
            Bal.   3092 × 38 = 117496
    July 9, Deliv. 1000
            Bal.   2092 × 25 = 52300
    Aug. 3, Deliv. 1500
            Bal.   592 × 13 =   7696
    " 16, Deliv. 592   30)249324

        Chests chargeable for 1mo., 83108

8310.8 × $0.03 = $249.32,4, cost of storage, Ans.

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8. 1857.   Bales.   da.   prod.
    Jan. 1, Rec.   2310 × 15 = 34650
    "   16, "   120
                    2430 × 16 = 38880
    Feb. 1, Rec.   300
                    2730 × 11 = 30030
    Feb. 12, Deliv. 1000
                    1730 × 17 = 29410
    Mar. 1, Deliv. 600
                    1130 × 33 = 37290
    Apr. 3, Deliv. 400 (carried forward).

Apr. 3, Deliv. $\underline{400}$ (brought forward).

$\overline{730} \times 7 = 5110$

Apr. 10, Deliv. 312

$\overline{418} \times 21 = 8778$

May 1, In store, 418ba. 30)$\overline{184148}$

Bales chargeable for 1 month, 6138

$6138 \times \$0.05. = \$306.90$, cost of storage up to **May 1.**

MISCELLANEOUS EXAMPLES.

(Page 333.)

1. The ratio of $1120, A's gain, to $3500, A's stock $= \frac{1120}{3500}$ $= \frac{8}{25}$.

   The ratio of $880, B's gain, to $2200, B's stock $= \frac{880}{2200} = \frac{10}{25}$.

   The ratio of $1200, C's gain, to $2500, C's stock $= \frac{1200}{2500}$ $= \frac{12}{25}$.

   $\frac{10}{25} - \frac{8}{25} = \frac{2}{25} =$ the gain of 2mo., B's time more than A's; $\frac{2}{25} :$ 2mo. :: $\frac{8}{25} :$ 8mo., A's time; $8 + 2 = 10$mo., B's time;

   $\frac{2}{25} :$ 2mo. :: $\frac{12}{25} :$ 12mo., C's time, Ans.

2. $540 + $660 = $1200; $540 ÷ 1200 = 45cts. on $1; $15000 × .45 = $6750, paid by the bankrupt; then, $15000 — $6750 = $8250, Ans.

3. $64.50 — $1.50 = $63, A's property tax; $50.50 — $1.50 = $49, B's property tax; $63 ÷ .007 = $9000, A's property; $49 ÷ .007 = $7000, B's property, Ans.

4. $20 + $30 + $150 = $200, total expenses; $200 — $50 = $150, raised by tax; $150 ÷ .03 = 5000, days of attendance, Ans.

5. Kimball's whole loss was made up *minus* a 5 per cent. tax on his share of the cargo; therefore, $7000 × .05 = $350, Ans.

6. $50 for 4mo. is $1 for 200mo.; and $100 for 8mo. is $1 for 800mo. If, then, $1 is to be paid in $200 + 800 = 1000$ months, $150 should be paid in $\frac{1}{150}$ of 1000mo. $= \frac{1000}{150}$ $= 6\frac{2}{3}$mo. It appears, then, that $150 is paid $6\frac{2}{3}$mo. before it is due. The question now is, In how much *less* than 10mo. should the $250 be paid? As we have before shown that $1 is to be paid in 1000mo., $250 should be paid in $\frac{1}{250}$ of 1000mo. $= 1000 \div 250 = 4$mo. We find, then, that the time in which B is to pay A is 4mo. *less* than it otherwise would have been, on account of A's making present payment. We therefore subtract 4mo. from 10, and find the true answer; thus $10 - 4 = 6$mo., Ans.

OPERATION.

$$50 \times 4 = 200$$
$$\underline{100 \times 8 = 800}$$
$$\overline{150} \quad 150)\overline{1000}(6\tfrac{2}{3}\text{mo.}$$
$$\underline{1000}$$

$$150 \times 6\tfrac{2}{3} = 1000 \div 250 = 4\text{mo.}$$
$$10 - 4 = 6\text{mo., Ans.}$$

(7.)

$$\tfrac{1}{3} = \tfrac{4}{12} \qquad\qquad \tfrac{4}{12} + \tfrac{3}{12} + \tfrac{2}{12} + \tfrac{3}{12} = \tfrac{12}{12}$$
$$\tfrac{1}{4} = \tfrac{3}{12} \times 6 = \tfrac{18}{12} \qquad \tfrac{18}{12} + \tfrac{16}{12} + \tfrac{36}{12} = \tfrac{70}{12}$$
$$\tfrac{1}{6} = \tfrac{2}{12} \times 8 = \tfrac{16}{12} \qquad \tfrac{70}{12} \div \tfrac{12}{12} = 5\tfrac{10}{12}\text{mo.} =$$
$$\tfrac{1}{4} = \tfrac{3}{12} \times 12 = \tfrac{36}{12} \qquad 5\text{mo. 25da., Ans.}$$

8. $2000 + his services are worth $800 per year; but $4000 + his services are worth $1100 per year; therefore, $2000 are worth $300 per year; hence his services without the $2000 are worth $800 − $300 = $500, Ans.

9.
$$\$100 \times 4 = 400$$
$$\$100 \times 8 = 800$$
$$\$200 \times 12 = 2400$$
$$\overline{\$400} \qquad \overline{3600}$$

$3600 \div 400 = 9$mo. for $400; $200 : 400 :: 9 : 18$mo. for $200; that is, $200 running 18mo. is equal to $400 running 9mo. If now 18 be divided into two parts,

one of which shall be twice as great as the other, we have 12
and 6; hence, if one note of $200 run 12mo., and the
other note of $200 run 6mo., it will be equivalent to
$200 in 18mo., or to $400 in 9mo.; therefore, in 6mo.
and 12mo., Ans.

10. The question involved in this problem is, In what time will
the interest of $360 be $3.78; $360 × .06 = $21.60,
interest of $360 for 1y.; then, 3.78 ÷ 21.60 will be the
time in years (Art. 360) = $\frac{378}{2160}$ = $\frac{7}{40}$y. = 2.1mo.
= 63da., Ans.

INSURANCE.

(ART. 445, p. 335.)

2. $896 × .12 = $107.52, Ans.

3. $17289 × .01¼ = 216.11¼, Ans.

4. $35000 × ¾ × .03¼ = $682.50; and $75000 × ⅝ ×
.02¼ = $1250; $1250 + $682.50 = $1932.50, the
amount of premium, Ans.

5. The loss was what the insurance cost; therefore, $3675 ×
.04⅞ = $179.15⅝, the premium; $179.15⅝ + $1, the
cost of the policy = $180.15⅝, Ans.

6. 690.90 ÷ 9870 = .07, Ans.

7. .04¼ + .03⅞ + .04⅜ + .05 + .05⅓ = .23⅓; $47600 ×
.23⅓ = $11007.50, Ans.

(ART. 446, p. 336.)

2. $26250 ÷ .87½ = $30000, Ans.

3. $6370 ÷ .98 = $6500, Ans.

4. In this problem the rate per cent. and interest are given, by
which to find the principal; then (Art. 358), $50 ÷ .01¼
= $4000, Ans.

5. As the policy is 10 per cent. of $600, the property must be
90 per cent. of $600; therefore $600 × .90 = $540,
Ans.

6. $1.00 - .03 = .97$; $1000 $\div$ .97 = $1030.92,7+.
   $1.00 - .05 = .95$; $1030.92,7 $\div$ .95 = $1085.18,7+.
   $1.00 - .06 = .94$; $1085.18,7 $\div$ .94 = $1154.45,4+.
   $1.00 - .07 = .93$; $1154.45,4 $\div$ .93 = $1241.34,8+.
   Ans.

LIFE INSURANCE.

(ART. 454, p. 340.)

2. $8000 $\times$ .0114 = $91.20, Ans.

3. $78000 $\times$ .0491 = $3829.80, Ans.

4. $2000 $\times$ .0088 = $17.60, Ans.

5. $12000 $\times$ .0373 = $447.60, the annual premium; the
   insured died in the third year, so that this premium was
   paid for three years; therefore, $447.60 $\times$ 3 = $1342-
   .80, and $12000 - $1342.80 = $10657.20, Ans.

6. 80y. — 50. = 30y., for which the annual premium was
   paid; $5000 $\times$ .046 = $230; $230 $\times$ 30 = $6900,
   the amount of premiums paid; therefore $6900 - $5000
   = $1900, Ans.

7. The number of years the premium is paid is 4;
   $4000 $\times$ $\frac{5.20}{100}$ = $208, the annual premium;
   The interest of $1 for 4y., and 3y., and 2y., and 1y., is re-
   spectively .24 + .18 + .12 + .06 = .60; and $208 $\times$
   .60 = $124.80, the amount of interest; $208 $\times$ 4 =
   $832, the amount of premium; $124.80 + $832 =
   $956.80; then, $4000 - $956.80 = $3043.20, Ans.

8. $10000 $\times$ .02 = $200, the annual premium.
   The last annual premium paid is not charged with interest;
   to shorten the work, we take the amount of $1 at com-
   pound interest from the table, for the periods of 11y., and
   10y., and 9y., etc.; these added together give $15.869-
   947, which is the sum of the amounts of an annual pre-
   mium of $1 for 11y.; a premium of $200 will be 200

16

times this = $ 15.869947 × 200 = $ 3173.9894; to
this add the premium for the 12th year, which is not
chargeable with interest; $ 3173.9894 + $ 200 = $ 33-
73.9894; and $ 10000 — $ 3373.9894 = $ 6626.01,
Ans.

NOTE. — The work of this problem might be much shortened by referring
to the table of Annuities later in the book.

CUSTOM-HOUSE BUSINESS.

(ART. 460, p. 343.)

2. $ 4350 × .08 = $ 348, Ans.

8. $ 7890 × .15 = $ 1183.50; $ 1183.50 × .19 = $ 224-
.865, Ans.

4. 986 × 4.84 × .24 = $ 1145.3376, Ans.

5. 112.50 ÷ 750 = 15 per cent., Ans.

6. 60 × .95 × 70   = 3990fr., value of champagne;
   36 × .95 × 35   = 1197fr., value of port;
   50 × (31 — 1) × 4 = 6000fr., value of sherry;
               11187

  11187 × .30 = 3356.1fr. duties; 3356.1 × .18$\frac{6}{10}$ =
  $ 624.2346, Ans.

7. $ 53.76 ÷ .08 = $ 672; 60 × (112 — 12) = 6000lb.;
  $ 672 ÷ 6000 = 11$\frac{1}{5}$ cts., Ans.

8. 40 × 63 × .98 = 2469.6gal. molasses charged with duty;
  24 × 400 × .90 = 8640lb. sugar charged with duty;
       2469.6 × 3 × 10 × .24 = $ 177.81
       8640 × 1 × 5 × .24 = 103.68
       260 × 2 × .08   = 41.60
       410 × 7 × .30   = 861.00
               $ 1184.09, Ans.

9. 270 × $ 50 = $ 13500; $ 16740 — $ 13500 = $ 3240;
  3240 ÷ 13500 = .24, Ans.

10. Whole invoice $20560

$$\overline{3000} \times .04 = \$120.00$$
$$4200 \times .08 = \;336.00$$
$$2100 \times .15 = \;315.00$$
$$\underline{6000}$$
$$\$\overline{15300}$$

Bal. of invoice $\overline{\$5260} \times .30 = \underline{1578.00}$
$$\$2349.00, \text{ Ans.}$$

11. $10 \times 40 \times \frac{5}{20} \times .24 = 24 \quad £.$

$200 \times \frac{4}{20} \times \qquad .19 = 7.6 \quad £.$

$100 \times \frac{2\frac{1}{2}}{20} \times \qquad .15 = 1.875£.$

$60 \times .04 \qquad\qquad = \underline{2.4 \quad £.}$

$\qquad\qquad 35.875£. \times 4.84 = \$173.635,$

[Ans.

12. $32 \times 32 = 1024$ yds.; $\$122.88 \div .24 = \$512$, invoice value of the cloth; $\$512 \div 1024 = \$0.50$ per yard; $\$512 + \$122.88$, the cost of duties, $+ \$40.96$, other charges, $= \$675.84$; $\$675.84 \div 1024 = \$0.66$, cost per yard, Ans.

COINS AND CURRENCIES.

(ART. 470, p. 348.)

3. $46£. 16s. 6d. = 46.825£.$; $46.825 \div \frac{2}{5} = \$117.06\frac{1}{4}$, Ans.

4. $1032 \times \frac{3}{8} = 387£.$, Ans.

5. $515.70 \div 4 = 128.925£.$; $128.925£. = 128£. 18s. 6d.$, Ans.

6. $160.50 \times .18\frac{6}{10} = \$29.853$, Ans.

7. $728.41 \div 4.84 = 150\frac{241}{484}£. = 150£. 9s. 11\frac{61}{121}d.$, Ans.

8. $12£. 12s. = 12.6£.$; $12.6 \div \frac{7}{30} = \$54$, Ans.

9. 128£. 18s. 6d. = 128.925£.; 128.925 × 4 = $515.70, Ans.

10. 740.45 × $0.75 = $555.33¾, Ans.

11. 46£. 16s. 6d. = 46.825£.; 46.825 ÷ ⅖ = $117.06¼; 117.06¼ × $\frac{3}{10}$ = 35.118¾£. = 35£. 2s. 4½d., Ans.

12. 151 × 1.12 = $169.12; 169.12 ÷ .10 = 1691.2 reals plate, Ans.

13. 1000 × 1.06 = $1060; 1060 ÷ 18$\frac{6}{10}$ = 5698$\frac{8}{8}$ francs, Ans.

14. 30 × 30 × 15 = 13500s. sterling = 675£.; 675 × 4.84 = $3267; 3267 ÷ 4 = 816.75£. = 816£. 15s. Canada currency, Ans.

15. 410 × .80 = $328; 328 ÷ .69 = 475$\frac{25}{69}$, Ans.

EXCHANGE.

(ART. **493,** p. **354.**)

2. $1950.50 × 1.00¼ = $1955.37, Ans.

3. $2000 × 1.005 = $2010, Ans.

4. $1744.40 ÷ .98 = $1780, face of draft, Ans.

5. $600 × .99 = $594; $594 — $6.30 (the interest of $600 for 63 days) = $587.70, Ans.

6. $9256.40 × 1.00⅜ = 9291.11; $9291.11 — $50.91 (the interest of $9256.40 for 33 days) = $9240.20, Ans.

7. Find the value of $1 at ⅜ per cent. premium, and interest off for 33 days at 6 per cent.; $1 × 1.00⅜ = $1.00375; $1.00375 — $0.0055 = $0.99825, the value of $1 by the conditions of the question; hence, as $0.99825 is to $1, so is $9240.20 to that sum of which $9240.20 is the value or yield; therefore, $9240.20 ÷ .99825 = $9256.39+, Ans.

8. The yield of $1, by the conditions of the question, is $0.982; then, $1190.184 ÷ .982 = $1212, Ans.

(Art. **499**, p. **357**.)

3. $3676 \div 5\frac{20}{100} = \$706.92\frac{4}{13}$, Ans.

4. $3000\frac{12}{18} \times \$0.35 = \$1050.21\frac{7}{8}$, Ans.

5. $81727.75 \div 4.867 = 16792.223\pounds. = 16792\pounds.$ 4s. $5\frac{1}{4}$d., Ans.

6. $79000 \times .186 = \$14694$, the par value of 79000 francs; $\$14694 - \$14400.12 = \$293.88$; $293.88 \div 14694 = .02$, Ans.

7. $5763.75 \times \$0.74 = \$4265.175$, Ans.

8. $550.66 \div 1.10 = 500\frac{6}{10}$ millreas $= 500$ millreas 600 reas, Ans.

9. $10000 \div .68 = 14705\frac{20}{68}$ thalers $= 14705$ thalers 26 groschen $5\frac{1}{17}$ pfennings, Ans.

10. $400\frac{12}{48} \times 1.06 = \$424.265$, Ans.

11. 300 thalers 20 groschen 0 pfennings $= 300.6\frac{11}{18}$ thalers; $300.6\frac{11}{18} \times .69 = \$207.47\frac{11}{18}$, Ans.

12. $5694 \div .40 = 14235$ rupees, Ans.

13. $615.60 \div .80 = 769.5$ ducats $= 769$ ducats 5 carlini, Ans.

14. $7300 \div 1500 = \$4.86\frac{2}{3}$; $4.86\frac{2}{3} - 4.44\frac{4}{9} = .42\frac{2}{9}$; $.42\frac{2}{9} \div 4.44\frac{4}{9} = .09\frac{1}{2}$, Ans.

15. $868\pounds.$ 17s. 6d. $= 868.875\pounds.$; $868.875 \times 23.60 = 20505.45$ francs $= 20505$ francs 45 centimes, Ans.

16. $2640 \div .18 = 14666.66\frac{2}{3}$ lire $= 14666$ lire $66\frac{2}{3}$ centesimi, Ans.

17. $17280 \div .186 = 92903\frac{7}{31}$ francs; $92903\frac{7}{31} - 88128 = 4775\frac{7}{31}$; $4775\frac{7}{31} \div 88128 = 5\frac{1925}{1743}$ per cent., Ans.

18. English money at $8\frac{1}{2}$ per cent. premium $= \frac{40}{9} \times 1.085 = 4\frac{34}{9} = \$4.82\frac{2}{3}$; $17000 \times 4.82\frac{2}{3} = \$81984.444$, Ans.

16*

ARBITRATION OF EXCHANGES.

(Art. 505, p. 361.)

(2.)

$$1m. = 5.95fr.$$
$$5.20fr. = \$1$$
$$\$3500 = ---m.$$

$$\frac{1 \times 5.20 \times 3500}{1 \times 5.95} = 3058.823\tfrac{9}{17} \text{ millreas} = 3058 \text{ millreas}$$

$823\tfrac{9}{17}$ reas, Ans.

(3.)

EXCHANGE DIRECT.

$\$4\tfrac{2}{8}$ = 1£.
2000£. = $9777.777

EXCHANGE THROUGH PARIS.

$1 = 5.20fr.
24fr. = 1£.
2000£. = $9211.538

EXCHANGE THROUGH HAMBURG.

$0.35 = 1 mar. bane.
13¾ m. b. = 1£.
2000£. = $9625.

Exchange through Paris is better than exchange direct by
$566.239; and better than exchange through Hamburg
by $413.462.

4. $5000.00
    12.50, ¼ per cent. discount added;
    ——————
   $5012.50, exchange value of $5000 at New Orleans, to be
            paid in New York;

$5012.50
   25.0625, ½ per cent. of $5012.50 subtracted;
   ——————
$4987.4375, exchange value of $5012.50 at St. Louis, or
            New Orleans. The exchange being made
   through New Orleans, $4987.4375 in St. Louis will pay a
   debt of $5000 in New York; $5000 × 1.015 = $5075
   = the amount that must be paid by direct exchange;
   therefore, $5075 — $4987.4375 = $87.56¼, Ans.

5. $4.84⅔ = 1£.

 1£. = $\frac{4.20}{1.7}$ thalers.

 9760th. = $6698.25+; $6698.25×1.005=$6731.74+

               [Ans.

6. $0.34   = 1 mark banco.

 2½ m. b. = 1 ruble.

 ——   = $650.

$$\frac{650}{.34 \times 2\frac{1}{2}} = 764\frac{12}{17} \text{ rubles} = 764 \text{ rubles } 70\frac{10}{17} \text{ kopecks,}$$

               [Ans.

7.   By the question, $1 on Boston = $1.00¼ on Philadelphia,
 by direct exchange; and $1 on Boston = $1 on Chicago,
 and $1 on Chicago = $0.98 on Philadelphia; conse-
 quently, $1 on Boston = $0.98 on Philadelphia, by cir-
 cuitous exchange; therefore, exchange on Boston through
 Chicago is better than direct exchange by the difference
 between $1.00¼ and $0.98 = 2¼ per cent., Ans.

8.   8 guilders = 1 Amsterdam ell,

 1 ell   = $\frac{27}{36} = \frac{3}{4}$yd., and   } Amsterdam.

 8 guilders = $3.20, and

 $3.20 = ¾yd., or $4.26⅔ =1yd.

 3½ thalers = 1 Berlin ell,

 1 ell   = $\frac{25\frac{1}{2}}{36} = \frac{17}{24}$yd., and } Berlin.

 3½ thalers = 3½ × .66 = $2.31,

 $2.31 = $\frac{17}{24}$yd., or 3.26\frac{2}{17}$ = 1yd.

 1£. = $4.86⅔; $\frac{15}{18}$ of $4.86⅔ = $3.65, } England.

 $3.65 = 1yd., bought in England.

Therefore, Berlin is better for the purchase than Amster-
 dam or England, by the difference between 3.26\frac{2}{17}$ and
 $4.26⅔, or $3.65 respectively.

(9.)

$$\$4.80 = 1\pounds.$$
$$1\pounds. = 25\tfrac{1}{4} \text{ francs.}$$
$$7000 \text{ francs} = \$\text{------}$$

$$\frac{4.80 \times 7000}{25\tfrac{1}{4}} = \$1330.69, \text{ Ans.}$$

(10.)

$$1\pounds. = 13\tfrac{1}{3} \text{ guilders.}$$
$$1g. = 2\tfrac{1}{5} \text{ francs.}$$
$$\text{------ francs} = 1000\pounds.$$

$$\frac{13\tfrac{1}{3} \times 2\tfrac{1}{5} \times 1000}{1} = 29333\tfrac{1}{3} \text{ francs};\ 29333\tfrac{1}{3} - 146\tfrac{2}{3}\ (\tfrac{1}{2} \text{ per}$$

cent. commission) $= 4186\tfrac{2}{3}$ whole gain; $4186\tfrac{2}{3} \div 25000$
$(1000\pounds. \times 25 = 25000$ francs$) = 16\tfrac{5}{7}\tfrac{6}{5}$ gain per cent., Ans.

ALLIGATION.

(Art. 506, p. 363.)

(2.)

$$\$1.00 \times 19 = \$19.00$$
$$.66 \times 40 = 26.40$$
$$.50 \times 11 = \underline{5.50}$$
$$70) \quad \$50.90$$
$$\overline{\$0.727\tfrac{1}{7}, \text{ Ans.}}$$

(3.)

$$22 \times 3 = 66$$
$$20 \times 3 = 60$$
$$6) \ \overline{126}$$
$$\overline{21}, \text{ Ans.}$$

(4.)

$$70 \times 20 = \$14.00$$
$$60 \times 15 = 9.00$$
$$40 \times 80 = \underline{32.00}$$
$$115) \quad \overline{55.00}$$
$$\overline{\$0.47\tfrac{1}{2}\tfrac{8}{3}, \text{ Ans.}}$$

(ART. **509**, p. **365**.)

(2.)

$$80\begin{cases} \left.\begin{array}{l}\text{1bu. at } 45,\text{ gain } 35 \\ \text{1bu. at } 75,\text{ gain } 5\end{array}\right\} = 40 \text{ gain.} \\ \underline{\left.\begin{array}{l}\text{1bu. at } 100,\text{ loss } 20 \quad = 20 \text{ loss}\end{array}\right.} \\ \text{1bu. at } 100,\text{ loss } 20 \quad = 20 \text{ loss} \end{cases}$$

$$\left.\begin{array}{l}= 20 \text{ loss} \\ = 20 \text{ loss}\end{array}\right\} = 40 \text{ loss.}$$

1bu. of barley, 1bu. of rye, and 2 of wheat, Ans.

(3.)

$$20\begin{cases} \left.\begin{array}{l}\text{1 of 24, loss } 4 \\ \text{1 of 23, loss } 3\end{array}\right\} = 7 \text{ loss.} \\ \left.\begin{array}{l}\text{1 of 19, gain } 1 \\ \text{1 of 15, gain } 5\end{array}\right\} = 6 \text{ gain.} \\ \overline{\text{1 of 19, gain } 1 \quad = 1 \text{ gain}} \end{cases}$$

$$\bigg\} = 7 \text{ gain.}$$

1 part of 24, 1 of 23, 1 of 15, and 2 of 19, Ans.

(4.)

$$75\begin{cases} \left.\begin{array}{l}\text{1 gal. at} \quad 0,\text{ gain } 75 \\ \text{1 gal. at} \quad 60,\text{ gain } 15\end{array}\right\} = 90 \text{ gain.} \\ \left.\begin{array}{l}\text{1 gal. at} \quad 80,\text{ loss} \quad 5 \\ \text{1 gal. at } 120,\text{ loss } 45 \\ \overline{\text{8 gal. at} \quad 80,\text{ loss } (8 \times 5) \; 40}\end{array}\right\} = 90 \text{ loss.} \end{cases}$$

1 gal. of water, 1 of 60 cts., 1 of 120 cts., and 9 of 80 cts, Ans.

(ART. **510**, p. **366**.)

(2.)

$$100\begin{cases} \text{20 at} \quad 0, 2000 \text{ gain} = 2000 \text{ gain.} \\ \left.\begin{array}{l}\text{1 at 175,} \quad 75 \text{ loss} \\ \text{1 at 125,} \quad 25 \text{ loss} \\ \overline{\text{19 at 175, 1425 loss}} \\ \text{19 at 125,} \quad 475 \text{ loss}\end{array}\right\} = 2000 \text{ loss.} \end{cases}$$

20 gallons of each price, Ans.

(3.)

$$190 \begin{cases} \left. \begin{array}{l} \text{1 at 180, gain} \quad 10 \\ \text{10 at 170, gain} \ 200 \end{array} \right\} = 210 \text{ gain.} \\ \left. \begin{array}{l} \text{4 at 220, loss} \ 120 \\ \underline{\text{1 at 200, loss} \quad 10} \\ \text{8 at 200, loss} \quad 80 \end{array} \right\} = 210 \text{ loss.} \end{cases}$$

9 bushels at $2.00; 1 bushel at $1.80, Ans.

(4.)

$$\left. \begin{array}{l} \text{1} \ \ \text{at} \ \ \text{8, gain} \ \ 4\frac{1}{2} \\ \text{3} \ \ \text{at} \ \ 9\frac{1}{4}, \text{gain} \ \ 9\frac{3}{4} \\ \text{4} \ \ \text{at} \ 10\frac{1}{2}, \text{gain} \ \ 8 \end{array} \right\} = 22\frac{1}{4} \text{ gain.}$$

$$\left. \begin{array}{l} \text{1} \ \ \text{at 13,} \ \ \text{loss} \ \ \ \frac{1}{2} \\ \text{6} \ \ \text{at } 13\frac{1}{2}, \text{loss} \ \ 6 \\ \underline{\text{1} \ \ \text{at 14,} \ \ \text{loss} \ \ 1\frac{1}{2}} \\ 4\frac{1}{2} \ \text{at 13,} \ \ \text{loss} \ \ 2\frac{1}{4} \\ \text{8} \ \ \text{at 14,} \ \ \text{loss} \ 12 \end{array} \right\} = 22\frac{1}{4} \text{ loss.}$$

Ans. 1lb. at 8 cts., 5½lb. at 13 cts., and 9lb. at 14cts.

(5.)

$$50 \begin{cases} \text{10 of 58, loss} \ \ 80 \ \ = 80 \text{ loss.} \\ \left. \begin{array}{l} \underline{\text{1 of 45, gain} \ \ 5} \\ \text{15 of 45, gain 75} \end{array} \right\} = 80 \text{ gain.} \end{cases}$$

Ans. 16 bushels of barley.

(Art. **511**, p. **367**.)

(2.)

$$15 \begin{cases} \left. \begin{array}{l} \text{1 at} \ \ 8, \text{gain} \ \ 7 \\ \text{1 at 10, gain} \ \ 5 \\ \text{1 at 12, gain} \ \ 3 \end{array} \right\} = 15 \text{ gain.} \\ \left. \begin{array}{l} \underline{\text{1 at 20, loss} \ \ 5} \\ \text{2 at 20, loss} \ 10 \end{array} \right\} = 15 \text{ loss.} \end{cases}$$

$1 + 1 + 1 + 3 = 6$; sugar at 8, 10, and 12 cents, will be each 1 part in 6; and sugar at 20 cents will be 3 parts in 6; $\frac{1}{6}$ of 200 = 33⅓lb. of 8, 10, and 12 cents; $\frac{3}{6}$ of 200 = 100lb. of 20 cents, Ans.

(3.)

$1.90 \begin{cases} 10\text{bu. at } 1.70, \text{ gain } 2.00 &= 2.00 \text{ gain.} \\ \dfrac{4\text{bu. at } 2.20, \text{ loss } 1.20}{8\text{bu. at } 2.00, \text{ loss } 0.80} \Big\} &= 2.00 \text{ loss.} \end{cases}$

$10 + 4 + 8 = 22$bu.; $48 - 22 = 26$bu. Then, as the gain on 1bu. at \$1.80 equals the loss on 1bu. at \$2.00, $\frac{1}{2}$ of $26 = 13$bu. $=$ the quantity at \$1.80, and $13 + 8 = 21$bu. $=$ the quantity at \$2.00, Ans.

(4.)

5lb. $= 60$oz.; 5oz. at 22 carats give a loss equal the gain on the 5oz. at 18 carats; and $5 + 5 = 10$oz.; $60 - 10 = 50$oz. Then,

$20 \begin{cases} 1\text{oz. at } 15, \text{ gain } 5 \\ 1\text{oz. at } 17, \text{ gain } 3 \end{cases} = 8 \text{ gain.} \\ 4\text{oz. at } 22, \text{ loss } 8 \quad = 8 \text{ loss.}$

$1 + 1 + 4 = 6$; $\frac{1}{6}$ of $50 = 8\frac{1}{3}$oz. $=$ the quantity at 15 carats, and at 17 carats; and $\frac{4}{6}$ of $50 = 33\frac{1}{3}$oz.; $33\frac{1}{3} + 5 = 38\frac{1}{3}$oz. $=$ the quantity at 22 carats, Ans.

(5.)

As 20 animals were bought for \$20, the average price of them was \$1.00.

$100 \begin{cases} \dfrac{2 \text{ sheep at } 400, \text{ loss } 600}{1 \text{ sheep at } 400, \text{ loss } 300} \Big\} = 900 \text{ loss.} \\ 1 \text{ lamb at } 50, \text{ gain } 50 \\ \dfrac{1 \text{ kid } \text{ at } 25, \text{ gain } 75}{\begin{array}{l} 1 \text{ kid } \text{ at } 25, \text{ gain } 75 \\ 14 \text{ lambs at } 50, \text{ gain } 700 \end{array}} \Bigg\} = 900 \text{ gain.} \end{cases}$

Ans. 3 sheep, 15 lambs, 2 kids.

MISCELLANEOUS EXAMPLES.

(Page 368.)

(1.)

$97\frac{1}{2} \begin{cases} 9 \text{ at } 80, \text{ gain } 157\frac{1}{2} \\ \dfrac{1 \text{ at } 120, \text{ loss } 22\frac{1}{2}}{6 \text{ at } 120, \text{ loss } 135} \Big\} = 157\frac{1}{2} \text{ loss.} \end{cases}$

$6 + 1 = 7$ men at \$1.20, Ans.

2. 5000 rix dollars 12 skillings = 5000¼ rix dollars; 5000¼
   × 1.06 = $5300.265, Ans.

3. If Sanford remits the amount, he buys a bill at Liverpool
   on New Orleans, and pays 10 per cent. premium; but if
   Lassale at New Orleans draws on Liverpool for the
   amount, he receives the amount at only 8 per cent. pre-
   mium; therefore the difference is the difference between
   1£. at 10 per cent. premium ($4.88⅔) and 1£. at 8 per
   cent. premium ($4.80) = $4.88⅔ — $4.80 = $0.08⅔,
   and this difference multiplied by 1500¾; then $0.08⅔ ×
   1500¾ = $133.40, Ans.

4. $1.56 is a gain of 20 per cent.; therefore $156 ÷ 1.20 =
   $1.30, the actual value of the mixture; hence,

$$130 \begin{cases} 17 \text{ at } 126, \text{ gain } 68. \\ 7 \text{ at } \text{—— loss } 68. \end{cases}$$

That is, 7 gallons cost so much that the loss, when sold at
130, was 68; then 68 ÷ 7 = 9⁵⁄₇cts. loss per gallon;
and $1.30 + $0.09⁵⁄₇ = $1.39⁵⁄₇, Ans.

5. 100 ounces 20 tari 10 graini = 100.68⅓ ounces; 100.68⅓
   × 2.40 = $241.64 ÷ .16 = 1510.25 lire = 1510
   lire 25 centesimi, Ans.

6.   20 U. S. gal.. = 1 eimer of Sweden.
             3 eimers of S. = 4 eimers of Trieste.
             24 eimers of T. = 9 ahms Danish.
             33 ahms D.   = 5 carri of Naples.
             12 carri of N. = —— U. S. gal.

$$\frac{20 \times 3 \times 24 \times 33 \times 12}{1 \times 4 \times 9 \times 5} = 3168 \text{gal.}; \quad 3168 \times \$0.80$$

(value of 1 ducat) = $2534.40, cost of 12 carri of wine.
Again,

             20 U. S. gal.   = 1 eimer of Sweden.
             3 eimers of S. = 4 eimers of Trieste.
             170 eimers of T. = —— U. S. gal.

$$\frac{20 \times 3 \times 170}{1 \times 4} = 2550\text{gal.}; \ 2550 \times \$0.84\tfrac{7}{8} \text{ (the value of}$$

1 florin 45 kreutzers in U.S. money) $= \$2164.31\tfrac{1}{4}$, cost of 170 eimers of Trieste; hence, 12 carri of wine will cost $\$2534.40 - \$2164.31\tfrac{1}{4} = \$370.08\tfrac{3}{4}$ *more* than 170 eimers of Trieste of wine, Ans.

7. 60s. = 3£.; 3 × 4.80 = $14.40; $14.40 ÷ 8.25 (bu. in 1qr.) $= \$1.74\tfrac{6}{11}$, the limited cost of a bushel delivered in Liverpool; this sum *less* the freight will be the limited cost of a bushel in Baltimore; 12d. $= \tfrac{12}{240}$ of 1£., of $4.80 = $0.24; then, $1.74\tfrac{6}{11} - \$0.24 = \$1.50\tfrac{6}{11}$, Ans.

8. 87 ÷ 1.16 = 75 cts., the actual value of the mixture. Since there is a difference of 12 cts. per pound in the two ingredients of the mixture, 11 pounds cancels eleven parts of the difference, and the 5 pounds cancels five parts of the difference; therefore, $\tfrac{11}{16}$ of 12 (the amount cancelled by the 11 pounds) must be deducted from 75 in finding the value of the 5 pounds per pound; 75 — $\tfrac{11}{16}$ of 12 = $66\tfrac{3}{4}$ cts., the value of each pound of the 5 pounds; 75 + $\tfrac{5}{16}$ of 12 = $78\tfrac{3}{4}$ cts., the value per pound of the 11 pounds, Ans.

INVOLUTION.

(Art. 515, p. 370.)		
1.	512.	5. $\tfrac{1}{243}$.
2.	1024.	6. 15625.
3.	$\tfrac{27}{64}$.	7. $16\tfrac{12144}{15625}$.
4.	$50\tfrac{46}{81}$.	8. 282475249.
		9. .000004100625.

17

	(Art. **516**, p. **370**.)	9.	96889010407
2.	2097152.	10.	2176782336.
3.	40353607.	11.	205891132094649+.
4.	60466176.	12.	363.691179+.
5.	281950621875.		
6.	$\frac{64}{729}$.	13.	$\left\{ \begin{array}{l} 157\frac{283}{1024}. \\ 1.800943+. \\ 116\frac{1225}{2401}. \end{array} \right.$
7.	22071204.		
8.	36372961.		

EVOLUTION.

EXTRACTION OF THE SQUARE ROOT.

	(Art. **525**, p. **376**.)	12.	$7\frac{1}{4}$.
5.	1856.	13.	$9\frac{3}{4}$.
6.	999.	14.	$19\frac{1}{19}$.
7.	$\frac{7}{12}$.	15.	1.4.
8.	15.3.	16.	81.
9.	$6\frac{1}{4}$.	17.	27.
10.	.027.	18.	64.
11.	4.16.	19.	3105671.

	(Art. **526**, p. **378**.)	4.	1.77482393+.
2.	2.5298+.	5.	19.3132079+.
3.	1.41421+.	6.	2.98831055+.

EXTRACTION OF THE CUBE ROOT.

(Art. **529**, p. **383**.)

3.	426.	9.	478.
4.	$\frac{5}{7}$.	10.	$1\frac{1}{7}$.
5.	4.39.	11.	$2\frac{9}{11}$.
6.	379.	12.	327.
7.	392.	13.	4968.
8.	.899.	14.	7583.

(Art. **530,** p. **383.**)

| 1. | 1.2599+. | 3. | 1.442249+. |
| 2. | 2.2239+. | 4. | 2.08008382301904+. |

EXTRACTION OF ANY ROOT.

(Art. **532,** p. **384.**)

(2.)	(3.)	(4.)
*3)998001	*2)262144	*3)43046721
3)332667	2)131072	3)14348907
*3)110889	2)65536	3)4782969
3)36963	*2)32768	3)1594323
*3)12321	2)16384	*3)531441
3)4107	2)8192	3)177147
*37)1369	*2)4096	3)59049
37)37	2)2048	3)19683
1	2)1024	*3)6561

$3 \times 3 \times 3 \times 37 = 999,$ *2)512   3)2187

[Ans.   2)256   3)729

2)128   3)243

*2)64   *3)81

2)32   3)27

2)16   3)9

*2)8   3)3

2)4   1

2)2 $3 \times 3 \times 3 \times 3 = 81,$

1   [Ans.

$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64,$ Ans.

	(5.)		(6.)
	*3)14348907		*5)11390625
	3)4782969		5)2278125
	3)1594323		'5)455625
	3)531441		5)91125
	3)177147		5)18225
	*3)59049		5)3645
	3)19683		*3)729
	3)6561		3)243
	3)2187		3)81
	3)729		3)27
	*3)243		3)9
	3)81		3)3
	3)27		1
	3)9		
	3)3		
	1		

$5 \times 3 = 15$, Ans.

$3 \times 3 \times 3 = 27$, Ans.

HORNER'S METHOD OF EXTRACTING ROOTS.

(ART. 533, p. 387.)

(8.)

```
0      0    41678648568(8467, Ans.
8      9    27                          (4.)
8      9    14678           0      0    48614208(852,
8     18    12304           8      9    27         [Ans.
6    2700   2369648         8      9    16614
8     876   2117786         8     18    15875
90   8076   251912568       6    2700   789208
     392    251912568       8     475   789208
14   846800                 90   8175
 4     6156                  5    500
98   352956                 95   367500
 4     6192    (Brought up.) 5    2104
1020 85914800     1032      100   369604
   6   72709        6         5
1026 35987509    10380      1050
   6       7        2
1082 (Carried up.) 10887    1052
```

(5.)

0	0	1.05(1.016397, Ans.
1	1	1
―	―	
1	1	50000
1	2	30301
―		―
2	30000	19699
1	301	18468
―	―	―
300	30301	1231
1	302	929
―	―	―
301	30603	302
1	18·	279
―	―	―
302	3078	23
1	18	22
―	―	―
303	3096	1
•	...	

(6.)

184528125(45, Ans.

4	16	64	256	1024
4	16	64	256	82128125
―	―	―	―	82128125
4	32	192	1024	
8	48	256	12800000	
4	48	384	3625625	
12	96	640000	16425625	
4	64	85125		
16	16000	725125		
4	1025			
200	17025			
5				
205				

17*

(7.)

```
    0        0         0          100(3.162278, Ans.
    8        9    ·    27          81
   ──       ──       ──          ──────
    8        9        27          190000
    8       18        81          113521
   ──       ──      ──────        ───────
    6       27       108000       7647900
    8       27         5521       7860080
   ──      ────      ──────       ───────
    9      5400       113521       287820
    8       121         5643       252672
  ───      ────      ───────       ──────
  120      5521       1191640       85148
    1       122         35040       25294
  ───      ────      ───────       ──────
  121      5648       1226680        9854
    1       123         35484        8853
  ───      ────      ───────       ──────
  122      5766       1262164        1001
    1        74          120·        1012
  ───      ────      ───────
  123      5840        126336
    1        74          120
  ───      ────      ───────
  124      5914        126456
   · ·       74            1·
           ────       ───────
           5988         12647
            · ·          · ·
```

(8.)

```
    0        0         0          0          6561(5.7995466,
    5       25       125        625          3125        [Ans.
   ──      ──      ────       ────          ─────────
    5       25       125        625          343600000
    5       50       375       2500          289192057
   ──      ──      ────      ─────          ─────────
   10       75       500       31250000      54407943
    5       75       750       10063151      49025772
   ──      ───     ──────      ────────      ────────
   15      150      1250000     41313151      5882171
    5      100       187593     11466854      5073030
   ──      ───     ───────     ────────      ───────
   20     25000     1437593     52780005      309141
    5      1799      200529      169308·       282755
  ───     ─────     ───────     ────────      ──────
  250     26799     1638122      5447808       26386
    7      1848      213808       171936       22624
  ───     ─────     ───────     ────────      ──────
  257     28647     1851930      5619244        3762
    7      1897        292·        1745·        3394
  ───     ─────     ──────      ────────      ──────
  264     30544      18812        563670         868
    7      1946        292         1745          389
  ───     ─────     ──────      ────────      ────
  271     32490      19104        565415          29
    7      · ·          292          10·
  ───               ──────      ────────
  278               19896         56551
    7                · · ·          10
  ───                            ───────
  285                             56561
    ·                              · · ·
```

## APPLICATION OF POWERS AND ROOTS.

(PAGE 392.)

1. $\sqrt{141376} = 376$, Ans.

2. $1760 \times 9 = 15840$; $\sqrt{15840} = 125.857+$ feet, Ans.

3. $144 \times 144 = 20736$; $64 \times 64 = 4096$; $20736 - 4096 = 16640$; $\sqrt{16640} = 128.99+$ feet, Ans.

4. $20 \times 20 = 400$; $16 \times 16 = 256$; $12 \times 12 = 144$; $400 + 256 + 144 = 800$; $\sqrt{800} = 28.28+$ feet, Ans.

5. $40 \times 40 = 1600$; $1600 \div 4 = 400$; $\sqrt{400} = 20$ rods, Ans.

6. $60 : 90 : : 66^2 = 4356 : 6534$; $\sqrt{6534} = 80.83+$ rods, Ans.

7. $2h. : 3h. : : (\frac{3}{4})^2 = \frac{9}{16} = .5625 : .84375$; $\sqrt{.84375} = .918+$inch, Ans.

8. $2^2 = 4 : 1.5^2 = 2.25 : : 50m. : 28m. 7\frac{1}{2}sec.$, Ans.

9. $4 \times 4 \times 3 = 48 : 6 \times 6 \times 2 = 72 : : 4h. : 6h.$, Ans.

10. $40 \times 90 = 3600$; $\sqrt{3600} = 60$, Ans.

11. $31\frac{1}{4} \times 20 = 625$; $\sqrt{625} = 25$lb., Ans.

12. $2 : 3 : : 2400 : 3600$; $\sqrt{3600} = 60$ trees in length. $3 : 2 : : 2400 : 1600$; $\sqrt{1600} = 40$ trees in breadth. $60 - 1 = 59 \times 7 = 413$; $40 - 1 = 39 \times 7 = 273$; $413 \times 273 = 112749$ square yards, Ans.

13. $50 \div 2 = 25$; $25^2 = 625$; $625 - 600 = 25$; $\sqrt{25} = 5$; $25 + 5 = 30$ years the older; $25 - 5 = 20$ years the younger, Ans.

14. $128 \times 128 = 16384$; $72 \times 72 = 5184 + 16384 = 21568$; $\sqrt{21568} = 146.86+$ miles, Ans.

15. $100 \times 100 = 10000$; $70 - 5 = 65$; $65 \times 65 = 4225$; $10000 - 4225 = 5775$; $\sqrt{5775} = 75.993420+$; $80 \times 80 = 6400$; $50 - 5 = 45$; $45 \times 45 = 2025$; $6400 - 2025 = 4375$; $\sqrt{4375} = 66.143782+$; 75.993420

$+ 66.143782 = 142.137202 \times 142.137202 = 20202$-
$.984192388804$; $70—50 = 20$; $20 \times 20 = 400$; 20202-
$.984192388804 + 400 = 20602.984192388804$; $\sqrt{20602}$-
$.984192388804 = 143.537306 +$ feet, Ans.

16. $\sqrt{16} = 4$; $44 + 4 = 48$; $48 \div 2 = 24$, the larger num-
ber; $44 — 4 = 40$; $40 \div 2 = 20$, the smaller, Ans.

17. Let A B represent the height of the tree, E the top of the
stump, C the point on which the top of the
tree will fall. As the tree will rest on the
stump, it is evident that A E will be equal
to C E. By drawing D E at right angles to
A C, it is evident that A D will be equal to
D C; that is, the line A C is bisected in D;
therefore we have two similar triangles, A B
C and A D E. This is evident from the fact
that each of these triangles has one right
angle, and that the angle A is common
to both triangles. Therefore, as A B is to A C, so is A D to
A E. If, then, we take A E from A B, the remainder, E B, will
be the answer required.
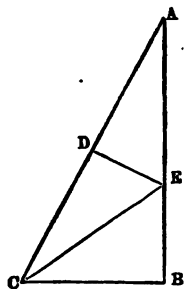
FIRST OPERATION.

A B is 80, and B C is 40; therefore A C will be equal to the
square root of the sum of the squares of A B and B C; that is,
$80^2 + 40^2 = 6400 + 1600 = 8000$; $\sqrt{8000} = 89.44 + =$
A C; A D will therefore be $44.72 +$. Then, as $80 : 89.44 : :$
$44.72 + : 49.9 +$; and if the fractions could all have been taken
into the above operation, the fourth term would have been 50
feet. Therefore, $80 — 50 = 30$ feet, Ans.

BY ALGEBRA.

Let x represent B E, $80 — x = $ C E, and $40 = $ C B. Then
$\overline{80 — x}^2 — x^2 = 40^2$; $6400 — 16x + x^2 — x^2 = 1600$. And
$160x = 6400 — 1600 = 4800$.

   $x = 30$ feet $= $ B E, Ans.

80 feet evidently equals the length of the part broken off and the stump together. The distance from the bottom of the tree to the point on which the top may fall, 40 feet, represents the base of a right-angled triangle, of which the part of the tree broken off is the hypothenuse, and the part left standing on the stump is the perpendicular. Then, according to ART. **553**, $40^2 = 1600 \div 80 = 20$; $80 - 20 = 60$; $60 \div 2 = 30$ feet, Ans.

18. $400 \times 400 = 160000$; $160 \times 160 = 25600$; $160000 - 25600 = 134400$; $\sqrt{134400} = 366.6 - 50 = 316.6$, Ans.

19. $70 \times 70 = 4900$; $40 \times 40 = 1600$; $30 \times 30 = 900$; $4900 - 1600 = 3300$; $\sqrt{3300} = 57.445+$; $4900 - 900 = 4000$; $\sqrt{4000} = 63.245 + 57.445 = 120.69+$ feet, Ans.

20. $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$: $1\frac{2}{10}$in. : : 45 0lb. : 45000lb., Ans.

21. 10 acres $= 435600$ square feet; $\sqrt{435600} = 660$ feet $=$ each side of the square plat; $660 \div 6 = 110 =$ the number of vines in each row, and also the number of rows, in the square order; $110 \times 110 = 12100 =$ the number of vines, in the square order.

In the quincunx order, the vines are disposed in the form of equilateral triangles; the distance between the rows, therefore, must equal the altitude of such triangles, having 6 feet for each of their equal sides. Hence, $6^2 - 3^2 = 27$; $\sqrt{27} = 5.196+$ feet $=$ the distance of the rows from each other; $660 \div 5.196 = 127 =$ the number of rows in the field; and the number of vines will be the same as in the square order. Therefore, $127 \times 110 = 13970 =$ the number of vines in the quincunx order; and $13970 - 12100 = 1870 =$ the number more of vines in the quincunx than in the square order, Ans.

In the square order no vine need be set *nearer* than 3 feet of the edge of the plat, and in the quincunx order nearer than $1\frac{1}{4}$ feet of the edge of the plat, and answer the conditions of the question.

22. $40 \times 40 = 1600$; $4 \times 4 = 16$; $1600 - 16 = 1584$;

$1584 \div 4 = 396$; $396 \times 3 = 1188$; $1188 + 16 = 1204$;
$\sqrt{1204} = 34.698$; $40 - 34.698 = 5.302$; $5.302 \div 2 =$
2.651in., the part A will grind off. $1204 - 396 = 808$;
$\sqrt{808} = 28.425$; $34.698 - 28.425 = 6.273$; $6.273 \div 2 =$
3.136+in., the part B will grind off. $808 - 396 = 412$;
$\sqrt{412} = 20.297$; $28.425 - 20.297 = 8.128$; $8.128 \div 2 =$
4.064in., part C grinds off. $412 - 396 = 16$; $\sqrt{16} = 4$;
$20.297 - 4 = 16.297$; $16.297 \div 2 = 8.148$+in., part D will
grind off.

Ans. A, 2.651in.; B, 3.136+in.; C, 4.064in.; D, 8.148+in.

23. $49\frac{5}{8}$ feet $= 49.625$ feet; $49.625 \times 144 = 7146$ square
inches; $1.5 \times 1.5 \times 2 = 4.5$; $7146 - 4.5 = 7141.5$; 7141.5
$\div 6 = 1190.25$; $\sqrt{1190.25} = 34.5$; $34.5 + 1.5 = 36$ inches,
Ans.

To understand the operation of this question, we will take six
small square pieces of board, each of the same dimensions, and
with these we will construct a cubic box; but, in so doing, we
shall find that we need two small cubes, each of which is of the
thickness of the board or plank used.

Now, if the box, after being completed, was a cube whose sides
measured 36 inches each, and if the board was $1\frac{1}{2}$ inches thick,
it would require 7146 square inches of the board to make the
box, as the pupil can readily perceive.

But our box was made of six square pieces of board and two
small cubes, each measuring, in the present case, $1\frac{1}{2}$ inches square.
If we deduct the contents of these two squares, $= 1.5 \times 1.5 \times$
$2 = 4.5$ square inches, from the superficial contents of the board,
$= 7146$ inches, we have $7146 - 4.5 = 7141.5$ square inches
remaining; and if we divide these inches by 6, we have the su-
perficial contents of one of the square boards of which we make
our box. Thus, $7141.5 \div 6 = 1190.25$. The square root of
this number, $\sqrt{1190.25} = 34.5$, will be the width of each board.
To this number we must add the thickness of the board, 1.5
inches, and we have the answer, $34.5 + 1.5 = 36$ inches.

24. $22\frac{31}{72}$ feet $= 22\frac{31}{72} \times 144 = 3242$ square inches; $2.5 \times$
$2.5 = 6.25$ inches; $3242 - 6.25 = 3235.75$; $3235.75 \div 7 =$

462.25 ; $\sqrt{}$ 462.25 = 21.5 ; 21.5 + 2.5 = 24 inches, the width of the box ; 24 ÷ 2 = 12 inches, the height ; 24 × 2 = 48 inches, the length.

From each of these numbers we subtract 5 inches, the thickness of the two sides of the box ; 12 − 5 = 7 ; 24 − 5 = 19 ; 48 − 5 = 43. The inside dimensions of the box will therefore be 7, 19, and 43 inches ; and its contents will therefore be 7 × 19 × 43 = 5719 cubic inches, Ans.

To understand the above operation, we will construct a box that shall be twice as wide as its height, and twice as long as its width. With the materials used to construct this box the pupil will find that he can construct a cubical box whose sides will measure half the length of the former box. And, if it be constructed as the box in question 44, there will be material remaining sufficient to make one of the sides, after deducting the square of the thickness of the plank. Thus, if our box was 12 inches high, 24 inches wide, and 48 inches long, and was made of a plank 2½ inches thick, it would require one whose superficial contents were 3242 inches. From this sum we deduct the square of the thickness of the plank, 2.5 × 2.5 = 6.25 inches; 3242 − 6.25 = 3235.75 inches. This sum, as we have before shown, is sufficient not only to construct the cubical box, but there will be sufficient remaining, wanting the square of the thickness of the plank, to make another, similar to one of the six squares of which we made the box. This will be evident, if the box be cut into two parts, and one of them placed on the other. Therefore, we divide 3235.75 by 7, because there are materials sufficient for 7 squares, and the quotient is 462.25. The square root of this number is the length of one of the squares of which the box is made. $\sqrt{}$ 462.25 = 21.5 inches. To this number we add the thickness of the plank, and we have the width of the required box. 21.5 + 2.5 = 24 inches, width of the box ; 24 ÷ 2 = 12 inches, height ; and 24 × 2 = 48 inches, length, Ans.

We have copied, by permission, from the *Massachusetts Teacher*, the following illustrations, with diagrams, of the 23d and 24th problems.

(23.)

Fig. 1.
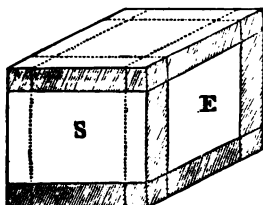
Suppose the 6 pieces of which the box is made to be so arranged (Fig. 1) that the edges of the top, bottom, and sides, will appear at the ends, as in the shaded parts about E; and that the edges of the top and bottom will appear at the sides, as in the shaded parts above and below S.

The external area of each face of the box may be considered as divided into, 1st, a large square, as at E; 2d, 4 strips, one on each side of the large square, having each a width equal to the thickness of the board ($1\frac{1}{2}$in.), and the same length as one side of the large square; and 3d, 4 small squares, each having, for the length of a side, the thickness of the board. In the 6 faces of the box, then, there are 6 large squares $+$ 24 strips $+$ 24 small squares.

But the shaded parts, though forming part of the external area of the sides and ends, do *not* form part of the side and end *pieces* of the box,* and are not to be reckoned as consuming any part of the surface of the board. If, then, from the large squares, strips, and small squares, that make up the whole external area of the box, we deduct 12 strips (2 for each side, and 4 for each end), and 16 small squares (4 for each side, and 4 for each end), we have left 6 large squares $+$ 12 strips $+$ 8 small squares $=$ the area of the board $= 49\frac{5}{8}$sq. ft. $= 7146$sq. in.

From this deduct the area of the 8 small squares ($(1\frac{1}{2}$in.$)^2 \times 8$ $= 18$sq. in.) and we have 6 large squares $+$ 12 strips $= 7146$sq. in. $- 18$sq. in. $= 7128$ sq. in. Divide this by 6, and we find one large square $+$ 2 strips $= \frac{7128}{6}$sq. in. $= 1188$ sq. in. This large square and the 2 strips may be arranged as in Fig. 2, which will be an exact square, if we fill the corner C ($1\frac{1}{2}$in.$)^2$

Fig. 2.
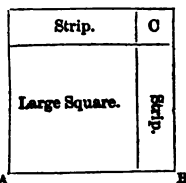
* Notice the difference between the side and side *piece*, and the end and end *piece*, of the box.

= $2\frac{1}{4}$sq. in. Then 1188sq. in. + $2\frac{1}{4}$sq. in. = the area of the square (Fig. 2) completed = $4\frac{161}{4}$sq. in. Then, $\sqrt{4\frac{161}{4}}$sq. in. = $\frac{69}{2}$in. = length of one side of the square, or the line A B. This, it is evident (see Fig. 1), is $1\frac{1}{2}$in. less than the length of a side of the box; therefore, $\frac{69}{2}$in. + $1\frac{1}{2}$in. = 36in., the length of one side.

(24.)

FIG. 3.
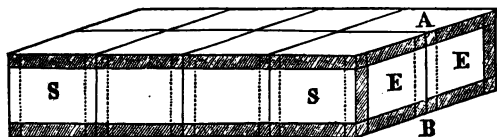
Let, in Fig. 3, the shaded parts about E E represent the edges of the top, bottom, and sides, at the end of the box; and the shaded parts above and below S S, the edges of the top and bottom, at the side. These shaded parts represent the thickness of the plank, and form part of the external area of the side and end of the box, but no part of the side and end *pieces.*

If we draw the line A B vertically across the middle of the end of the box, the whole end is divided into 2 equal squares; but, taking away the shaded portions, it is evident that the end *piece* is not divided by this line into squares, since we have diminished each square of the end in one direction by twice the thickness of the plank, and in the other by only once its thickness.

To get two equal squares in the end *piece*, let there be marked off on each side of A B, by dotted lines, two strips, each as wide as the plank is thick. We may now suppose the *end* of the box (not the end piece) to be divided into, 1st, 2 large squares, E and E; 2d, 8 strips, each equal in length to the side of a *large square*, and in width to the thickness of the plank ($2\frac{1}{2}$in.); and 3d, 8 small squares, each having its side equal in length to the thickness of the plank. Since the box is twice as wide as it is high, and twice as long as wide, the area of an end is doubled in a side, and quadrupled in the top or bottom. Then an end, a side, and the top or bottom, contain 7 times as many large squares, strips, and small squares, as an end; that is, 14 large squares + 56

18

strips + 56 small squares; and both ends, both sides, and the top
and bottom, or the whole external area of the box, = 28 large
squares + 112 strips + 112 small squares.

But these exceed the area of the 6 *pieces* of which the box is
made, by the strips and small squares in the edges of the top and
bottom at the sides, and of the top, bottom, and sides, at the ends.
Referring to Fig. 3, and noticing that the strips marked off by
dotted lines are part of the pieces in which they are found, and
not to be deducted, we see that we must deduct, for each side, 8
strips and 16 small squares; and for an end, 6 strips and 8 small
squares; or 28 strips and 48 small squares for both sides and both
ends. Subtracting these from the large squares, strips, and small
squares of the whole external area of the box, we have 28 large
squares + 84 strips + 64 small squares = the area of the 6
pieces = $22\frac{7}{12}$sq. feet. = 3242sq. in. From this deduct the 64
small squares ($(2\frac{1}{2}$in.$)^2 \times 64 = 400$sq. in.), and we have 28
large squares + 84 strips = 3242sq. in. — 400sq. in. = 2842
sq. in. Divide by 28, and we find 1 large square + 3 strips =
$2\frac{84}{28}$sq. in. = $2\frac{9}{3}$sq. in.

This large square and the three strips may
be arranged as in Fig. 4; namely, a strip
and a half on each of two adjacent sides of
the *large square*. This will be an exact
square if we fill out the corner C, which
contains a square each side of which is one
and a half times the width of a *strip*
($2\frac{1}{2}$in. $\times 1\frac{1}{2} = \frac{15}{4}$in.). Adding this small

FIG. 4.

$\frac{1}{2}$ Strip.	
Strip.	C
Large Square.	Strip.	$\frac{1}{2}$ Strip.

D ———————————— F

square ($(\frac{15}{4}$in.$)^2 = \frac{225}{16}$sq. in.), we have $2\frac{9}{3}$sq. in. + $\frac{225}{16}$sq.
in. = $18\frac{49}{16}$sq. in. = area of the square Fig. 4, completed. Then
$\sqrt{18\frac{49}{16}}$sq. in. = $4\frac{3}{4}$in. = length of a side of the square, or the
line D F.
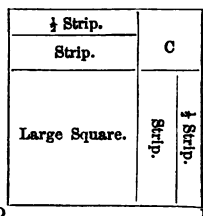
Comparing this with Fig 3, we see it wants half the thickness
of the plank ($2\frac{1}{2}$in. $\div 2 = 1\frac{1}{4}$in.) of being the external height
of the box. Then, $4\frac{3}{4}$in. + $1\frac{1}{4}$in. = $4\frac{3}{4}$in. = 12in. = external
height of the box; 12in. $\times$ 2 = 24in. = external width of the
box; and 24in. $\times$ 2 = 48in. = external length of the box.

Subtract from each of these dimensions twice the thickness of the plank, and we have 12in. — 5in. = 7in. = internal height of the box; 24in. — 5in. = 19in. = internal width of the box; 48in. — 5in. = 43in. = internal length of the box, and the product of the internal dimensions = 5719 cubic in. = the contents of the box.

25. $3 \times 3 \times 3 = 27 : 6 \times 6 \times 6 = 216 :: 4$lb. : 32lb., Ans.

26. $1 \times 1 \times 1 = 1 : 3.5 \times 3.5 \times 3.5 = 42.875 ::$ \$120 : \$5145, Ans.

27. 5ft. 10in. = 70in; 10ft. $4\frac{3}{5}$in. = 124.6in.; $70 \times 70 \times 70 = 343000$; $124.6 \times 124.6 \times 124.6 = 1934434.936$; $343000 : 1934434.936 :: 180$lb. : 1015.1+lb., Ans.

28. 2lb. : 2000lb. :: $4 \times 4 \times 4 = 64$in. : 64000; $\sqrt[3]{64000} = 40$in. = 3ft. 4in. high;
    2lb. : 2000lb. :: $3 \times 3 \times 3 = 27$in. : 27000in.; $\sqrt[3]{27000} = 30$in. = 2ft. 6in. wide;
    2lb. : 2000lb. :: $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64} : \frac{1000}{64}$; $\sqrt[3]{\frac{1000}{64}} = \frac{10}{4} = 2\frac{1}{2}$in. thick, Ans.

29. $12096 \div 56 = 216$; $\sqrt[3]{216} = 6$; $56 \times 6 = 336$; $336 \times 6 = 2016$; 336 and 2016, mean proportions;
    Then, $56 : 336 :: 2016 : 12096$, Ans

30. $5 \times 5 \times 5 = 125$ft. : $20 \times 20 \times 20 = 8000$ft. :: 1cwt. : 64cwt., Ans.

31. $6 \times 6 \times 6 = 216$ft. : $10 \times 10 \times 10 = 1000$ft. :: 1da. : 4.629+da., Ans.

32. $6 \times 6 \times 6 = 216$ft. : $8 \times 8 \times 8 = 512$ft. :: 600lb. : 1422.2+lb., Ans.

33. $5 \times 5 \times 5 = 125$; $125 \div 4 = 31.25$; $125 - 31.25 = 93.75$; $\sqrt[3]{93.75} = 4.542+$; $5 - 4.542 = .45+$in., the first woman's share. $93.75 - 31.25 = 62.50$; $\sqrt[3]{62.50} = 3.968$; $4.542 - 3.968 = .57+$in., share of the second woman. $62.50 - 31.25 = 31.25$; $\sqrt[3]{31.25} = 3.149$; $3.968 - 3.149 = .82+$in., third woman's share. $3.149+$in., fourth woman's share.

34. If Wells have 100 per cent., Rowe will have 108 per cent.,
and Northend 110 per cent. But Pierce has 10 per cent.
more than Rowe; therefore he will have $108 \times 1.10 =$
118.8 per cent. $118.8 + 108 + 100 + 110 = 436.8$;
5 tons $= 100$cwt.

Then $436.8 : 118.8 :: 100$cwt. $: 27\frac{54}{273}$cwt. for Pierce.

And $436.8 : 108 \quad :: 100$cwt. $: 24\frac{198}{273}$cwt. for Rowe.

And $436.8 : 100 \quad :: 100$cwt. $: 22\frac{244}{273}$cwt. for Wells.

And $436.8 : 110 \quad :: 100$cwt. $: 25\frac{50}{273}$cwt. for Northend.

To find the feet in height that each must take, we adopt the
following rule, and say, As the relative value of all their
shares is to the relative value of each share, so is the cube
of the height of the pyramid or stack to the cube of the
height of each man's part of the stack. But we are to
compute from the top of the stack each time, and then
subtract as in the following process:

$16 \times 16 \times 16 = 4096$ft., cube of the height of the stack.
$436.8 : 118.8 :: 4096 : 1114.02197$; $\sqrt[3]{1114.02197}$
$= 10.366+$ft. in height for Pierce.

$436.8 : 118.8 + 108 = 226.8 :: 4096 : 2126.76923076$;
$\sqrt[3]{2126.76923076} = 12.859+$ from the top of the
stack. From this number we subtract the height of
Pierce's stack, $12.859 - 10.366 = 2.493$ft. for the
height of Rowe's stack.

$436.8 : 118.8 + 108 + 100 = 326.8 :: 4096 : 3064-$
$.49816849$; $\sqrt[3]{3064.49816849} = 14.525$ from the top of
the stack. From this we subtract the height of the other
two stacks; $14.525 - 12.859 = 1.666$ft. for the height
of Wells' stack.

If from the height of the stack, 16ft., we subtract the last
root, we have the remaining height of the stack for North-
end; thus, $16 - 14.525+ = 1.474$ft., height of North-
end's stack.

NOTE. — The decimals in the answer to the above question will vary
according to the degree of accuracy required.

ARITHMETICAL PROGRESSION.

(Art. **557**, p. **396.**)

2. $\dfrac{39-3}{19-1} = 2\text{y., Ans.}$

3. $\dfrac{45-5}{11-1} = 4\text{m., Ans.}$

(Art. **558**, p. **397.**)

2. $\dfrac{51-7}{4} + 1 = 12\text{da., Ans.}$

3. $\dfrac{103-1}{2} + 1 = 52\text{w., Ans.}$

(Art. **559**, p. **398.**)

2. $\dfrac{100+4}{2} \times 17 = 884,$ Ans.

3. $320 \times 30 = 9600 =$ rods in 30m.; but there will be one more stone in this distance than there are rods, because there will be a stone at each end of the 30 miles. The man must travel 2 rods to bring the first stone to the basket, and 60 miles and 2 rods to bring the last stone; wherefore the following formula:

$$\dfrac{\overline{19202+2} \times 9601}{2} = 92188802\text{rd.} = 288090\text{m. 2rd., Ans.}$$

(Art. **560**, p. **398.**)

2. $\dfrac{\overline{45+3} \times \overline{45-3+2}}{2 \times 2} = 528,$ sum of the series, Ans.

3. $\dfrac{\overline{618 \times 6} \times \overline{618-6} + 12}{2 \times 12} = \$162.24,$ Ans.

18*

(ART. **561**, p. **399**.)

2. $\dfrac{100010000 \times 2}{10000} - 20000 = 2$, Ans.

3. $\dfrac{528 \times 2}{22} - 3 = \$45$; $\dfrac{45 - 3}{22 - 1} = \$2$ difference, Ans.

(ART. **562**, p. **400**.)

2. $\dfrac{49 - 4}{6 - 1} = 9$, common difference.

As there are 6 terms, the third and fourth are required; $4 + 9 + 9 = 22$, the third term; $22 + 9 = 31$, the fourth term, Ans.

3. $\dfrac{30 - 20}{7 - 1} = 1\frac{2}{3}$, the common difference.

$20 + 1\frac{2}{3} = 21\frac{2}{3}$; $21\frac{2}{3} + 1\frac{2}{3} = 23\frac{1}{3}$; $23\frac{1}{3} + 1\frac{2}{3} = 25$; $25 + 1\frac{2}{3} = 26\frac{2}{3}$; $26\frac{2}{3} + 1\frac{2}{3} = 28\frac{1}{3}$. $21\frac{2}{3}$, $23\frac{1}{3}$, 25, $26\frac{2}{3}$, and $28\frac{1}{3}$, Ans.

GEOMETRICAL PROGRESSION.

(ART. **565**, p. **401**.)

2. $5 \times 3^{7-1} = 3645$, seventh term, Ans.

3. $\dfrac{72}{3^{6-1}} = \dfrac{8}{27}$, first term, Ans.

4. $\dfrac{885735}{3^{12-1}} = \dfrac{885735}{177147} = 5$, twelfth term;

$5 \times 3 = 15$, eleventh term; $15 \times 3 = 45$, tenth term, [Ans.

5. $5 \div (\frac{1}{3})^{7-1} = 5 \div \frac{1}{729} = 3645$, first term, Ans.

6. $50 \times 1.06^{5-1} = 63.123848$, last term, Ans.

7. $2 \times 2^{30-1} = \$10737418.24$, Ans.

8. $\$160 \times 1.06^{7-1} = \$226.96305796096$, Ans.

9. $\$300 \times 1.05^{9-1} = \$443.23,6+$, Ans.

10. $\$100 \times 1.06^{31-1} = \$574.34911729132501162641063323$-$10802645846357252196069357387776$, Ans.

(Art. **566**, p. **402**.)

5. $\dfrac{1.06^4 - 1}{1.06 - 1} \times 50 = 218.7308$, sum of the series, Ans.

6. $\dfrac{2^{40} - 1}{2 - 1} \times 10 = \$109951162777.50$, Ans.

7. $\tfrac{2}{3} \div (1 - \tfrac{1}{2}) = \tfrac{2}{3} \div \tfrac{1}{2} = \tfrac{4}{3} = 1\tfrac{1}{3}$, Ans.

8. By examining this question, we find there have been 21 deposits. The amount of the last deposit is $\$10.60$, the $\$10$ being on interest only one year. The last but one is $\$11.236$. The last but two is $\$11.91016$. The last but three is $\$12.6247696$, and so on. Thus we have a regular geometrical series, the ratio of which is 1.06, the first term $\$10.60$, the number of terms 21, to find the sum of all the series.

$$\dfrac{1.06^{21} - 1}{1.06 - 1} \times 10.60 = \$423.92\overset{2}{2} +, \text{ Ans.}$$

9. $.008 = \tfrac{8}{1000}; \ .000497133 = \tfrac{497133}{1000000000}$, the first term; $\tfrac{1}{1000000}$, the ratio.

$1 - \tfrac{1}{1000000} = \tfrac{999999}{1000000}; \ \tfrac{497133}{1000000000} \div \tfrac{999999}{1000000} = \tfrac{497133000000}{999999000000000} = \tfrac{497133}{999999000}; \ \tfrac{497133}{999999000} + \tfrac{8}{1000} = \tfrac{8878125}{999999000} = \tfrac{83}{9768}$, Ans.

10. $10 \div (1 - \tfrac{9}{10}) = 10 \div \tfrac{1}{10} = 100$ miles, Ans.

(Art. **567**, p. **403**.)

2. $512 \div 1 = 512; \ \sqrt[9]{512} = 2$ ratio, Ans.

3. $\dfrac{1328600 - 5}{1328600 - 885735} = 3$ ratio, Ans.

4. $\sqrt[12-1]{\dfrac{2048}{1}} = 2$ ratio. $\dfrac{2048 \times 2 - 1}{1} = \$4095$ debt,

[Ans.

(ART. **568**, p. **404.**)

2. $128 \div \frac{1}{2} = 256$; $\sqrt{256} = 4$ ratio; $\frac{1}{2} \times 4 = 2$; $2 \times 4 = 8$; $8 \times 4 = 32$. 2, 8, and 32, Ans.

3. $2187 \div 3 = 729$; $\sqrt{729} = 3$; $3 \times 3 = 9$; $9 \times 3 = 27$; $27 \times 3 = 81$; $81 \times 3 = 243$; $243 \times 3 = 729$. 9, 27, 81, 243, and 729, Ans.

(ART. **569**, p. **405.**)

2. $20480 \div 5 = 4096 = 4^6$; $6 + 1 = 7$, Ans.

3. $2048 \div 1 = 2048 = 2^{11}$; $11 + 1 = 12$ months, Ans.

ANNUITIES.

(ART. **574**, p. **408.**)

2. $\dfrac{1.06^5 - 1}{1.06 - 1} \times \$500 = \$2818.546+$, Ans.

3. $\dfrac{1.05^9 - 1}{1.05 - 1} \times \$80 = \$882.125$, Ans.

4. $\dfrac{1.035^{15} - 1}{1.035 - 1} \times \$1000 = \$19295.68$, Ans.

5. $\dfrac{1.035^6 - 1}{1.035 - 1} \times \$30 = \$196.50$, Ans.

6. $\dfrac{1.015^{22} - 1}{1.015 - 1} \times \$600 = \$5404.295$, Ans.

(ART. **575**, p. **409.**)

2. $\$6.801692 \times 100 = \$680.169$, Ans.

3. $\$5.786373 \times 200 = \$1157.274$, Ans.

4. $\$13.331709 \times 500 = \$6665.854$, Ans.

5. $\$7.360087 \times 500 = \$3680.043$, Ans.

6. $\$12.561102 \times 80 = \$1004.888$, Ans.

(ART. **576**, p. **409.**)

2. $963 ÷ .06 = $16050, Ans.

3. $6335 ÷ .07 = $90500, Ans.

4. $1200 ÷ .05 = $24000, Ans.

(ART. **577**, p. **410.**)

2. $11.469421 — $6.801692 = $4.667729; $4.667729 ×
   350 = $1633.705, Ans.

3. $13.590326 — $6.732745 = $6.857581; $6.857581 ×
   70 = $480.03, Ans.

4. $240 ÷ .06 = $4000, the present worth of $240 in
   perpetuity.
   $16.617546, the present value of an annuity of $1, due
   100 years hence.
   $16.617546 × 240 = $3988.211;
   $4000 — $3988.211 = $11.789, Ans.

   or,

   $1.06¹⁰⁰ = $339.302084+.
   $4000 ÷ 339.302084 = $11.789,

(ART. **578**, p. **411.**)

2. $3680.04 ÷ 7.360087 = $500, Ans.

3. $882.12,5 ÷ 11.026564 = $80. Ans.

4. $279 ÷ 6.975319 = $40, Ans.

PERMUTATIONS AND COMBINATIONS.

(ART. **581**, p. **412.**)

2. $1 × 2 × 3 × 4 × 5 × 6 = 720$ changes, Ans.

3. $1 × 2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 × 10 = 3628800$
   days, Ans.

4. $1 × 2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 × 10 × 11 × 12$
   $= 479001600$ changes. 365d. 5h. 49m. = 525949 min-

utes; $479001600 \div 10 = 47900160$ minutes; 47900160
$\div 525949 = 91y. \ 38801m. = 91y. \ 26d. \ 22h. \ 41m.$, Ans.

5. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12$
$\times 13 \times 14 \times 15 \times 16 \times 17 \times 18 \times 19 \times 20 \times 21$
$\times 22 \times 23 \times 24 \times 25 \times 26 = 40329146112660563$-
5584000000 changes, Ans.

(ART. **582**, p. **412**.)

2. $6 \times 5 \times 4 \times 3 = 360$ changes, Ans.

3. $26 \times 25 \times 24 \times 23 \times 22 \times 21 = 165765600$, Ans.

(ART. **583**, p. **413**.)

2. $\dfrac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7} = \dfrac{720}{6} = 120$, Ans.

3. $\dfrac{100 \times 99 \times 98 \times 97 \times 96 \times 95 \times 94 \times 93 \times 92 \times 91}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10} =$
$\dfrac{6281565095555294720}{3628800} = \$173103094564.40$, Ans.

ANALYSIS BY POSITION.

(ART. **587**, p. **414**.)

(2.)

Supposed number, 120

$\frac{1}{3}$ of 120 $=$ $\overline{40}$

$\frac{1}{4}$ of 120 $=$ $\underline{30}$

                    $\overline{70}$

Result found, 50

$50 : 60 : : 120 : 144$, Ans.

(3.)

Supposed number, 36

$\frac{1}{2}$ of 36 $=$ 18

$\frac{1}{3}$ of 36 $=$ 12

$\frac{1}{4}$ of 36 $=$ $\underline{9}$

Result found, $\overline{75}$

$75 : 125 : : 36 : 60$, Ans.

(4.)

C's assumed age,   20

B's age $=$ 60

A's age $=$ $\underline{120}$

                    $\overline{200}$

$200 : 140 : : 20 : 14$, C's age;

$14 \times 3 = 42$, B's age; $42 \times 2 = 84$, A's age, Ans.

(5.)

Assumed sum, $\$400$

                    $\underline{1.60}$

Amount for 10 yr., $\$640.00$

$640 : 560 : : 400 : 350$, Ans.

(Art. **588,** p. **415.**)

2. First suppose each laid out $500; then

4)$500	$500
125	225
625 = A's money.	275 = B's money.

Then, by the question, $2 \times 275 =$ $550 should be A's money, but A's money is $625; therefore $625 — $550 = $75 is the first error.

Again, we suppose each laid out $800; then

4)$800	$800
200	225
1000 = A's money.	575 = B's money.

Then, by the question, $2 \times$ $575 =$ $1150 should be A's money, but A's money is $1000; therefore $1150 — $1000 = $150+, second error; then, by the rule, 75 + 150 : 800 — 500 : : 75 : 100; and 100 + 500 = 600, • the sum of dollars each invested, Ans.

3. We first suppose the age of the youngest to be 10. Then, by the question, the age of the next older will be 14, and the next 18, and the oldest will be 22. But, by the supposition, the age of the oldest was twice the age of the youngest, that is, $2 \times 10 = 20$; but the age of the oldest is 22, therefore the first error is 22 — 20 = 2— too small.

Again, we suppose the age of the youngest to be 16. Then the age of the next older will be 20, and the age of the next 24, and the age of the oldest will be 28. But, by the supposition, the age of the oldest was twice the age of the youngest, that is, $2 \times 16 = 32$; but the age of the oldest is 28; therefore, the second error is 32 — 28 = 4+ too large; then, by the rule, 2 + 4 : 16 — 10 : : 4 : 4; 16 — 4 = 12, the age of the youngest; $12 \times 2 =$ 24, age of the oldest; 24 — 4 = 20, age of the second; 20 — 4 = 16, age of the third son, Ans.

4. We suppose the value of the first horse to be $20. Then 20 + 50 = $70 will be twice the value of the second horse; that is, the value of the second horse will be $35. Then, by the question, 35 + 50 = $85 will be three times the value of the first horse; that is, the first horse will be worth $28⅓. But, by the supposition, he should be worth but $20; therefore 28⅓ — 20 = $8⅓— is the first error. Again, we suppose the value of the first horse to be $32. Then 32 + 50 = $82 will be twice the value of the second horse; that is, the second horse will be worth $41. Then, by the question, 41 + 50 = $91 will be three times the value of the first horse; that is, the first horse will be worth $30⅓. But, by the supposition, he should be worth $32; therefore, 32 — 30⅓ = 1⅔, the second error; then, 8⅓ + 1⅔ : 32 — 20 :: 8⅓ : 10; and 10 + 20 = $30, value of the first horse; $\frac{20+50}{2}$ = $40, value of the second horse, Ans.

5. If we suppose the time to be 3 o'clock, it will be 9 hours to midnight, and ⅔ of 3 hours will be equal to $\frac{8}{33}$ of 9 hours; but ⅔ of 3 hours is ⅔ × ⅓ = $\frac{6}{3}$ = 2 hours, and $\frac{8}{33}$ of 9 hours is $\frac{8}{33}$ × ⅑ = $\frac{72}{33}$ = 2$\frac{2}{11}$ hours; therefore the first error will be 2$\frac{2}{11}$ — 2 = $\frac{2}{11}$—. Again, we suppose the time to be 4 o'clock. Then the time to midnight will be 8 hours; therefore, by the question, ⅔ of 4 hours will be equal to $\frac{8}{33}$ of 8 hours, but ⅔ of 4 hours is ⅔ × ¼ = ⅔ = 2⅔ hours; and $\frac{8}{33}$ of 8 hours is $\frac{8}{33}$ × ⅛ = $\frac{64}{33}$ = 1$\frac{31}{33}$ hours; therefore, the second error will be 2⅔ — 1$\frac{31}{33}$ = $\frac{8}{11}$+. Then, by the rule, $\frac{2}{11}$ + $\frac{8}{11}$: 4 — 3 :: $\frac{20}{110}$: $\frac{22}{110}$ = ⅕; 3 + ⅕ = 3⅕ hours, the time after noon = 12 minutes past 3, Ans.

6. We first suppose their income to be $360 each. Then, as A saves $\frac{1}{12}$ of his income, he will spend $360 ÷ 12 = $30; $360 — $30 = $330 annually. As B spends $100 per annum more than A, he will spend $330 + $100 = $430 each year; and in 10 years he will spend

$10 \times \$430 = \$4300$. As his income, by the supposi-
tion, is only $360 per annum, it would be in ten years
$10 \times \$360 = \$3600$. His debt would therefore be
$4300 — 3600 = \$700$. But by the question it was
only $600; therefore the first error will be $700 —
$600 = \$100—$ too small.

Again, we suppose their income to be $300 annually. Then
as A saves $\frac{1}{12}$ of his, he will spend $\$300 \div 12 = \$25$;
$300 — \$25 = \$275$ annually; and B, by the question,
will spend $\$275 + \$100 = \$375$, and in 10 years he
will spend $10 \times \$375 = \$3750$. B's income by the
question will be only $10 \times \$300 = \$3000$; therefore
his debt will be $\$3750 — \$3000 = \$750$. But, by the
question, it was only $600, therefore the second error will
be $\$750 — \$600 = \$150—$ too small; then (the errors
being both made by results too small, the difference is
taken), $150 — 100 : 360 — 300 : : 100 : 120$; $120 +
360 = \$480$, Ans.

7. It appears, by the tenor of the question, that 90 must be so
divided into two parts that, if the larger part be multi-
plied by 60, and the smaller part by 80, the difference of
the products shall be 3300. We therefore suppose the
larger part to be 60, and the smaller part 30, and pro-
ceed according to the rule, thus: $60 \times 60 = 3600$;
$30 \times 80 = 2400$. The difference between these prod-
ucts is $3600 — 2400 = 1200$, which is $3300 —1200 =
2100$ *less* than it should be. Again, we divide 90 into
the two parts 80 and 10, and multiply them by 60 and
80 as before, and find the result as follows: $80 \times 60 =
4800$; $10 \times 80 = 800$; $4800 — 800 = 4000$, which is
$4000 — 3300 = 700$ *more* than it should be. Hence we
have the statement, $700 + 2100 : 80 — 60 : : 700 : 5$;
$80 — 5 = 75$, the days of labor; and $90 — 75 = 15$,
days of idleness, Ans.

8. First suppose 48 the weight of the fish; as the body weighs

as much as the head and tail, its weight will be half of
the whole fish; half of 48 = 24; but 24 + 15 + (15 + $\frac{1}{8}$
of 24) = 58; therefore, 58 — 48 = 10, first error.

Again, suppose the weight of the fish to be 60; but 30 +
15 + (15 + $\frac{1}{8}$ of 30) = 65 for the weight of the fish
by the conditions of the question; therefore 65 — 60 = 5,
second error.   Then, 10 — 5 : 60 — 48 : : 10 : 25;
therefore, 25 + 48 = 72, weight of the fish, Ans.

9. Assume 300 pounds to be growing on each acre (any other
number would answer as well); then 3$\frac{1}{3}$ acres will con-
tain 1000 pounds.   We now suppose the weekly increase
to be 9 pounds; then 3$\frac{1}{3}$ acres will, in four weeks, pro-
duce 120 pounds.

$$3\tfrac{1}{3} \times 300 = 1000$$
$$9 \times 3\tfrac{1}{3} \times 4 = 120$$

Amount of 3$\frac{1}{3}$ acres in 4 weeks = 1120lbs.

Having found that 12 oxen will eat 1120 pounds of grass in
4 weeks, we wish to know how many pounds 21 oxen
would eat in 9 weeks, which, by the following process, we
find to be 4410 pounds.   Thus,

$$\left. \begin{array}{c} 12 : 21 \\ 4 : 9 \end{array} \right\} : : 1220 : 4410\text{lbs}.$$

But we find the amount of the grass of 10 acres in 9 weeks
to be 3810 pounds.   Thus,

$$300 \times 10 = 3000$$
$$9 \times 10 \times 9 = \underline{810}$$
$$3810\text{lbs}.$$

But, by the supposition, 21 oxen in 9 weeks would eat 4410lbs.

The first error will therefore be   —600

We next suppose the weekly increase to be 18 pounds per
acre; therefore the amount of the grass of 3$\frac{1}{3}$ acres in 4
weeks will be 1240 pounds.   Thus,

$$300 \times 3\tfrac{1}{3} = 1000$$
$$18 \times 3\tfrac{1}{3} \times 4 = \underline{240}$$
$$1240\text{lbs}.$$

And the amount of 10 acres in 9 weeks would be 4620lbs.
Thus,

$$10 \times 300 = 3000$$
$$10 \times 18 \times 9 = 1620$$
$$\overline{4620\text{lbs.}}$$

But, by the last supposition, we find that 21 oxen in 9 weeks
would eat 4882½ pounds. Thus,

$$\left. \begin{array}{r} 12 : 21 \\ 4 : 9 \end{array} \right\} : : 1240 : 4882\tfrac{1}{2}\text{lbs.}$$

By subtracting 4620 from 4882½ pounds, we find   4882½
  the second error, to be —262½.   4620
                                                      ―――――――
                                                          —262½

Hence, the statement, $600 - 262\tfrac{1}{2} : 18 - 9 : : 600 : 16$;
and $16 + 9 = 25$ pounds, weekly increase.

Having assumed 300 pounds to the acre, and found the
  weekly increase to be 25 pounds to the acre, which is 8⅓
  per cent., we now proceed to find the amount of the pro-
  duce of 24 acres for 18 weeks. Thus,

$$24 \times 300 = 7200$$
$$25 \times 18 \times 24 = 10800$$

Amount of 24 acres for 18 weeks $= \overline{18000\text{lbs.}}$

$$3\tfrac{1}{3} \times 300 = 1000$$
$$3\tfrac{1}{3} \times 4 \times 25 = 333\tfrac{1}{3}$$

Actual amount of 3⅓ acres for 4 weeks $= \overline{1333\tfrac{1}{3}\text{lbs.}}$

The question now is, If 12 oxen eat 1333⅓ pounds of grass
  in 4 weeks, how many oxen will eat 18000 pounds in 18
  weeks? Then,

$$\left. \begin{array}{l} 1333\tfrac{1}{3}\text{lbs.} : 18000\text{lbs.} \\ 18 \text{ weeks} : 4 \text{ weeks} \end{array} \right\} : : 12 : 36 \text{ oxen, Ans.}$$

10. By trial the required number is found to be between 26 and
    and 27, which numbers may be assumed successively.
    Then, by extracting the square root of 26, trebling the
    root, and taking the result from 26, we get 10.703; 11—

10.703 = .297, first error; proceeding the same way with 27, the result is 11.412; 11.412 — 11 = .412, second error; then, by the rule, .297 + .412 : 27 — 26 : : .297 : .42; and .42 + 26 = 26.42, the first approximation. Assume now 26.42 for the number; then, 26.42 — 3 times its square root = 10.999883269; 11 — 10.999-883269 = .000116731, first error; next assume 26.421; 26.421 — 3 times its square root = 11.000591445; 11.000591445 — 11 = .000591445, second error. Then, .000116731 + .000591445 : 26.421 — 26.42 : : .000116-731 : .0001648; and 26.42 + .0001648 = 26.4201648, Ans.

SCALES OF NOTATION.

(Art. 592, p. 418.)

(2.)	(3.)
2)37	3)1000000
2)18 1	3)333333 1
2)9 0	3)111111 0
2)4 1	3)37037 0
2)2 0	3)12345 2
1 0	3)4115 0
Ans. 100101.	3)1371 2
	3)457 0
	3)152 1
	3)50 2
	3)16 2
	3)5 1
	1 2

Ans. 1212210202001, in the ternary scale; and

9)1000000
9)111111 1
9)12345 6
9)1371 6
9)152 3
9)16 8
1 7

In the nonary scale, 1783661, Ans.

(4.)
12)476897
12)39741 5
12)3311 9
12)275 *e*
12)22 *e*
1 *t* Ans. 1*tee*95.

(ART. **593**, p. **419**.)

(2.)
234
5
13
5
69, Ans.

(8.)
21122
8
7
3
22
8
68
8
206, Ans.

19*

(4.)	(5.)
100101	13579
2	12
─	─
2	15
2	12
─	─
4	185
2	12
─	─
9	2227
2	12
──	─────
18	26733
2	
──	
37, Ans.	

11)26733
11)2430   3
11)220   t
11)20   0
.1   9
Ans. 190t3.

(6.)
123454821
6
─
8
6
──
51
6
───
310
6
────
1865
6
─────
67167
6
──────
403004
6
───────
2418025

12)2418025
12)201502   1
12)16791   t
12)1399   3
12)116   7
9   8
Ans. 9873t1.

(ART. **594**, p. **420**.)

	(1.)			(2.)
	45324502			2483
	25405534			589
	$\overline{115134440}$, the sum, $\Big\}$ Ans.			$\overline{1t\,985}$
	15514524, the diff.,			18502
				11184
				$\overline{13122t\,5}$, Ans.

(3.)

589)1184323(2483, Ans.

$e\,56$

$\overline{22t\,3}$

$1te\,0$

$\overline{3e\,32}$

$39t\,0$

$\overline{1523}$

1523

$\overline{}$

(4.)

$1\dot{1}\dot{1}\dot{2}\dot{2}\dot{4}\dot{4}\dot{1}$(2405, Ans.

4

$\overline{44)\dot{3}\dot{1}2}$

304

$\overline{5205)42441}$

42441

DUODECIMALS.

(ART. **598**, p. **421**.)

(1.)			
121ft.	3′	9″	
105ft.	11′	8″	
80ft.	0′	6″	
15ft.	10′	0″	4‴
Ans. 323ft.	1′	11″	4‴

(2.)		
462ft.	4′	9″
307ft.	9′	1″
Ans. 154ft.	7′	8″

3. 92ft. 0′ 6″ — 21ft. 9′ 10″ = 70ft. 2′ 8″ ; and 70 ft. 2′ 8″ + 19ft. 10′ 8″ 6‴ = 90ft. 0′ 11″ 6‴, Ans.

(ART. **600**, p. **422**.)

2. 48ft. 6′ × 24ft. 3′ = 1176sq. ft. 1′ 6″, Ans.

3. 20ft. + 14ft. 6′ = 34ft. 6′, × 2 = 69ft. × 10ft. 4′ = 713ft. ;
   3ft. 2′ × 6ft. × 2 = 38ft.; 4ft. 4′ × 4ft. = 17ft. 4′, +
   38ft. = 55ft. 4′ ; 713ft. — 55ft. 4′ = 657ft. 8′ ; 657ft.
   8′ ÷ 9 = 73$\frac{2}{27}$ square yards, Ans.

4. 53ft. 6′ × 10ft. 3′ × 2ft. = 1096ft. 9′, Ans.

5. 6ft. 8′ + 5ft. 9′ + 4ft. 6′ + 3ft. 10′ = 20ft. 9′, × 3ft. 5′
   × 4 = 283sq. ft. 7′, Ans.

6. 97ft. 9′ × 3ft. 6′ = 342ft. 1′ 6″, ÷ 4 = 85$\frac{51}{256}$ cord ft., ÷
   8 = 10$\frac{1}{2}\frac{7}{56}$ cords, Ans.

7. 100ft. × 6ft. 11′ = 691ft. 8′ ÷ 4 = 172$\frac{11}{12}$ cord ft. ÷ 8 =
   21$\frac{8}{8}$ cords, Ans.

(Art. 601, p. 423.)

2. 834sq. ft. 3′ ÷ 17ft. 9′ = 47ft., Ans.

3. 18ft. 9′ × 3 = 56ft. 3′ ; 84ft. 4′ 6″ ÷ 56ft. 3′ = 1ft. 6′,
   Ans.

NOTE. — The thickness of the plank, 3′, should not be regarded as 3′ in working the problem, but simply as 3 times the thickness of board measure.

4. 792ft. 6′ 9″ 2‴ ÷ 12ft. 7′ 8″ = 62ft. 8′ 6″, Ans.

MISCELLANEOUS EXAMPLES.

(Page 424.)

1. As $52.50 is the average of salary, $52.50 — $20 = $32.50,
   average of increase; $32.50 × 2 = $65, whole increase;
   $65 + 20 = $85, salary received last month = last
   term; then, (Art. 558) $\frac{85-20}{5}$ + 1 = 14 months, Ans.

2. 20ft. + 16ft. 6′ = 36ft. 6′ × 2 = 73ft. × 9ft. 6′ = 693ft.
   6′ ; 20ft. × 16ft. 6′ = 330ft.; 330ft. + 693ft. 6′ =
   1023ft. 6′ × 3 = 3070ft. 6′ ÷ 9 = 341yd. 1ft. 6′ ;
   341yd. 1ft. 6′ — 90yd. = 251yd. 1ft. 6′, Ans.

3. (Art. 565.) $(1\frac{1}{2})^{10}$ = $\frac{59049}{1024}$; $\frac{59049}{1024}$ × 1024 = $59049,
   the share of the eldest, Ans.

4. (Art. 583.) $\frac{20\times19\times18\times17\times16\times15\times14\times13\times12\times11}{1\times2\times3\times4\times5\times6\times7\times8\times9\times10}$

   [= $1847.56, Ans.

5. 340ft. ÷ 3ft. 9′ = 90ft. 8′ = 30yd. 0ft. 8′, Ans.

6. By Table (p. 406) the amount of $1 for 6 years is $6.97-5319; therefore, $700 × 6.975319 = $4882.72, Ans.

7. (1.) We first suppose the time to be $60\frac{1}{2}$ seconds after 12 o'clock. The hour-hand then will have passed $\frac{60.5}{43200}$ of the distance from 12 to 12 again, and the second-hand will have passed once round and $\frac{1}{120}$ of another time. The difference between these two numbers is $\frac{1}{120} - \frac{60.5}{43200} = \frac{298}{86400}$. The minute-hand will have passed $\frac{60.5}{3600}$ of the distance from 12, and the difference between this number and $\frac{1}{120}$ is $\frac{60.5}{3600} - \frac{1}{120} = \frac{732}{86400}$. We now find the difference between this last number and $\frac{299}{86400}$; $\frac{732}{86400} - \frac{298}{86400} = \frac{123}{86400}$—, first error, too small.

We next suppose the time to be 61 seconds after 12 o'clock. The hour-hand then will have passed $\frac{61}{43200}$ of the distance from 12 to 12, and the second-hand will have passed once round and $\frac{1}{60}$ of another time. The difference of these two numbers is $\frac{1}{60} - \frac{61}{43200} = \frac{659}{43200}$. We now find the difference between the second-hand and minute-hand. The minute-hand has moved in 61 seconds $\frac{61}{3600}$ of the distance from 12 to 12; the difference between $\frac{61}{3600} - \frac{1}{60} = \frac{12}{43200}$. We next find the difference between this last number and $\frac{659}{43200}$; $\frac{659}{43200} - \frac{12}{43200} = \frac{647}{43200} = \frac{1294}{86400}+$, second error. As the denominators of our errors are the same number, we may reject them in the operation; for when fractions have a common denominator their values are as their numerators. $133 + 1294 : 61 - 60.5 :: 133 : \frac{665}{1427}$; and $\frac{665}{1427} + 60.5 = 60\frac{780}{1427}$ seconds, Ans.

(2.) Suppose the time to be $61\frac{1}{2}$ seconds after 12 o'clock. Then the hour-hand will have moved $\frac{61.5}{43200}$ of the distance from 12 o'clock to 12 again, and the minute-hand will have moved $\frac{61.5}{3600}$ of this distance, and the second-hand will have moved once round and $\frac{1.5}{60}$ again. The difference between $\frac{61.5}{43200}$ and $\frac{61.5}{3600}$ is $\frac{676.5}{43200}$. The difference between $\frac{1.5}{60}$ and $\frac{61.5}{3600}$ is $\frac{3420}{43200}$. We then find the dif-

ference between $\frac{678.5}{43200}$ and $\frac{344.0}{43200} = \frac{334.5}{43200}$—, the first error.

We then suppose the time to be 62 seconds after 12 o'clock. The hour-hand will then have moved from 12 o'clock $\frac{62}{43200}$ of the distance to 12 again, and the minute-hand have moved $\frac{62}{3600}$ of the distance, and the second-hand will have gone once round and $\frac{2}{60}$ of the distance again. The difference between $\frac{62}{3600}$ and $\frac{62}{43200}$ is $\frac{682}{43200}$. The difference between $\frac{2}{60}$ and $\frac{62}{3600}$ is $\frac{626}{43200}$. The difference between $\frac{626}{43200}$ and $\frac{682}{43200}$ is $\frac{14}{43200}$+, second error. $334.5 +$ $14 : 62 - 61.5 :: 14 : \frac{14}{697}$; and $62 - \frac{14}{697} = 61\frac{683}{697}$ seconds, Ans.

(3.) We will first suppose the time to be 59 seconds after 12 o'clock; the hour-hand will then have advanced $\frac{59}{43200}$ of the distance from 12 o'clock to 12 again, and the second-hand will be within $\frac{1}{60}$ of the distance to 12; therefore, the whole distance between the hour-hand and the second-hand will be $\frac{1}{60} + \frac{59}{43200} = \frac{779}{43200}$; and the distance between the hour-hand and minute-hand will be $\frac{59}{3600} - \frac{59}{43200} = \frac{649}{43200}$, and the difference between $\frac{779}{43200}$ and $\frac{649}{43200} = \frac{130}{43200}$—, the first error.

Again, we will suppose the time to be 58 minutes after 12 o'clock. The distance then between the second-hand and hour-hand will be $\frac{2}{60} + \frac{58}{43200} = \frac{1498}{43200}$. The distance between the hour-hand and minute-hand will be $\frac{58}{3600} - \frac{58}{43200} = \frac{638}{43200}$, and $\frac{1498}{43200} - \frac{638}{43200} = \frac{860}{43200}$— will be the second error. $860 - 130 : 59 - 58 :: 130 : \frac{13}{73}$; and $59 + \frac{13}{73} = 59\frac{13}{73}$ seconds, Ans.

MENSURATION.

(Art. 613, p. 426.)

1. $18 \times \frac{12}{2} = 108$, Ans.

2. $15.6 + 9.2 + 10.4 = 35.2$ feet; $35.2 \div 2 = 17.6$ feet; $17.6 - 15.6 = 2.00$; $17.6 - 9.2 = 8.4$; $17.6 - 10.4$

$= 7.2$; $17.6 \times 2 \times 8.4 \times 7.2 = 2128.896$; $\sqrt{2128}$-
$.896 = 46.139+$ feet, Ans.

3. $40 \times \frac{15}{2} = 300$, Ans.

4. $336 \div 3 = 112$, each side of the field; $112 \div 2 = 56$, half
the base; $112^2 - 56^2 = 9408$; $\sqrt{9408} = 97$, the per-
pendicular height, very nearly; $112 \times \frac{97}{2} = 5432$ rods
$= 33$ acres 152 rods, Ans.

(ART. 621, p. 427.)

1. $15 \times 2 = 30$, Ans.

2. $128 \times 48 = 6144$, Ans.

3. $12 \times 8 = 96$, Ans.

4. $358 \times 194 = 69452$, Ans.

5. $693 \times 693 = 480249$ft.; $480249 \div 272\frac{1}{4} = 1764$ rods;
$1764 \div 160 = 11$ acres 4 rods, Ans.

6. $40 \times 40 = 1600$; $20 \times 20 \times 2 = 800$; $1600 - 800 =$
800, Ans.

7. $\sqrt{3600} = 60$ yards; $3600 \div 2 = 1800$ yards; $\sqrt{1800}$
$= 42.427+$ yards; 60 yards $- 42.427$ yards $= 17.573$
yards; $17.573 \div 2 = 8.78+$ yards, Ans.

(ART. 622, p. 428.)

1. $\frac{75+33}{2} \times 20 = 1080$sq. ft., Ans.

2. $\frac{786+473}{2} \times 986 = 620687$ links; $620687 \div 625$ (the links
in a square rod) $= 993$ rods 3 yards $= 6$ acres 33 rods
3 yards, Ans.

(ART. 623, p. 428.)

1. Drawing the diagonal divides the garden into two triangles,
with sides 328, 598, 298, and 598, 456, 572.

328	$612 - 328 = 284$, rem.
598	$612 - 598 = 14$, rem.
298	$612 - 298 = 314$, rem.
2)1224	
612, half sum.	

598	$813 - 598 = 215$, rem.
456	$813 - 456 = 357$, rem.
572	$813 - 572 = 241$, rem.

2)1626

    813, half sum.

$612 \times 284 \times 14 \times 314 = 764059968$; $\sqrt{764059968} = 27641.63$; $813 \times 215 \times 357 \times 241 = 15038837415$; $\sqrt{15038837415} = 122632.97$; $27641.63 + 122632.97 = 150274.6$ sq. ft. $= 3$ acres 1 rood 31 rods 29 yards 3.85 feet, Ans.

2. $\frac{2.82}{2} \times 17.56 = 77.4396$ square chains, area of one triangle;

    $\frac{7.72}{2} \times 17.56 = 67.8694$ square chains, area of the other triangle;

    $77.4396 + 67.8694 = 145.309$ square chains, the area of the whole field; $145.309 \div 10$ (10 square chains make 1 acre) $= 14.5309$ acres $= 14$ acres 2 roods 5 rods, nearly, Ans.

(Art. **626**, p. **429**.)

1. $\frac{172.05}{2} \times 250 \times 5 = 107531.25$ square feet, Ans.
2. $\frac{308.305}{2} \times 356 \times 6 = 329269.74$ yd., Ans.
3. $60^2 \times 4.828427 = 17382.3372$ yards $= 3$ acres 2 roods 14 rods 19 yards, nearly, Ans.
4. $243^2 \times 7.694209 = 454335.34724 = 10$ acres 1 rood 28 rods 24 yards 6.34724 feet, Ans.

(Art. **633**, p. **430**.)

1. $144 \times 3.141592 = 452.389248$, Ans.
2. $7964 \times 3.141592 = 25019.638688$, Ans.
3. $512 \times 2 \times 3.141592 = 3216.9984$ feet $= 4$ furlongs 34 rods 5 yards 1 foot, Ans.

(Art. **634**, p. **430**.)

1. $1043 \times .318309 = 331.997$, Ans.

2. $25000 \times .318309 = 7957.74$, Ans.

3. $50 \times .318309 = 15.91549$, Ans.

(Art. **635**, p. **430**.)

1. $761^2 \times .785398 = 454840.475158$, Ans.

2. 1 mile $= 320$ rods; $320 \times 3 = 960$; $960^2 \times .785398 = 723822.7968$ square rods $= 4523.89+$ acres, Ans.

3. $1284^2 \times .079577 = 131195.098512$ square yards $= 27$ acres 17 rods 0.848512 square yards, Ans.

4. $16.9\frac{9}{2} \times 13.2\frac{2}{2} = 22477$ inches $= 17$ yards 3 feet 13 inches, Ans.

(Art. **636**, p. **430**.)

1. $79 \times 4\frac{7}{2} = 1856.5$, Ans.

2. The area of a circle of which the radius is $25 = 50^2 \times .785398$; then, $360° : 26° : : 50^2 \times .785398 : 141.8$ sq. ft., Ans.

### (Art. **637**, p. **431**.)

1. $360° : 49.25° : : 24^2 \times .785398 : 61.889$, area of sector A B C E; $\sqrt{12^2 - 5^2} = 10.908 =$ perpendicular D E; $10.908 \times 5 = 54.54$, area of triangle A C E; $61.889 - 54.54 = 7.35$, nearly, area of the segment A B C A, Ans.

2. By the second rule, $\dfrac{15 \times 24 \times 2}{3} + \dfrac{15^3}{24 \times 2} = 310.3125$ square rods $= 1$ acre 3 roods 30 rods 9.4 yards, Ans.

(Art. **638**, p. **431**.)

1. The first step in the working of this problem is to find the height of the segments lying on each side of the zone; the heights are elements necessary in calculating their areas. Let H F B C be the zone; the radii A C, A B, A F, A H, are each 12 feet;

20
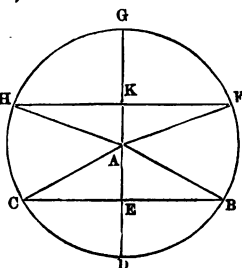
$A H = 12 ; 12^2 = 144 ;$

$H K = \dfrac{20.8}{2} = 10.4 ; 10.4^2 = 108.16.$

$144 - 108.16 = 35.84 ; \sqrt{35.84} = 5.9 = A K ; 12 - 5.9 = 6.1 = G K,$ the height of the segment whose cord is 20.8 ; then, $\dfrac{20.8 \times 6.1 \times 2}{3} + \dfrac{6.1^3}{20.8 \times 2} = 90.03,$ area of the segment H G F H ;

$A C = 12 ; 12^2 = 144 ;$

$C E = \dfrac{23.25}{2} = 11.625 ; 11.625^2 = 135.14 ; 144 - 135.14 = 8.86 ;$

$\sqrt{8.86} = 2.97 = A E ; 12 - 2.97 = 9.03 = D E,$ the height of the segment whose cord is 23.25 ; then $\dfrac{23.25 \times 9.03 \times 2}{3} + \dfrac{9.03^3}{23.25 \times 2} = 155.79,$ area of the segment C D B C ;

$24^2 \times .7854 = 452.39,$ area of the whole circle ;

$155.79 + 90.03 = 245.82,$ area of the two segments ;

$\overline{206.57},$ area of the zone, Ans.

2. Let 10, the radius, be the hypothenuse of a right-angled triangle, and 8, half the cord, be the base ; then, $10^2 - 8^2 = 36 ; \sqrt{36} = 6,$ the perpendicular from the centre to the cord ; $10 - 6 = 4,$ height of each segment ; then, by the rule (Art. **637**), $\dfrac{16 \times 4 \times 2}{3} \times \dfrac{4^3}{16 \times 2} = 44.66,$ area of each segment ; $44.66 \times 2 = 89.32,$ area of both segments ; $20^2 \times .785 =$

$314,$ area of the circle ;

$89.32,$ area of the segments.

$\overline{224.68},$ or 224.7, nearly,

area of zone, Ans.

(ART. **639**, p. **431**.)

1. By rule, Art. **637**, $\dfrac{72 \times 30 \times 2}{3} + \dfrac{30^3}{72 \times 2} = 1627.5$ sq. ft.

$\dfrac{72 \times 20 \times 2}{3} + \dfrac{20^3}{72 \times 2} = 1015.5$ sq. ft.

Ans. 612 sq. ft.

(ART. **640**, p. **431**.)

1. $(20 + 10) \times (20 - 10) \times .7854 = 235.62$ sq. yd., Ans.
2. $(157 + 128) \times (157 - 128) \times .7854 = 6491.331$ sq. yd.
   $= 1$ acre 1 rood 14 rods 17 yards 7.4 feet, Ans.

(ART. **641**, p. **431**.)

1. $50 \times .886227 = 44.31135$ rods, Ans.
2. $360 \times .282094 = 101.55$ rods, Ans.
3. $10000 \times .282094 = 2820.94$ rods, Ans.

(ART. **642**, p. **432**.)

1. $44.31135 \times 1.12838 = 50.000041113$ rods, Ans.

(ART. **643**, p. **432**.)

1. $30 \times .866025 = 25.98$ inches, Ans.
2. $5000 \times .275664 = 1378.320$ feet, Ans.
3. $80 \times .275664 = 22.05$ inches, Ans.

(ART. **644**, p. **432**.)

1. $30 \times .707106 = 21.21$ inches square, Ans.
2. $80 \times .707106 = 56.56848$ feet, Ans.
3. $5000 \times .225079 = 1125.395$ rods, Ans.
4. $100 \times .225079 = 22.5$ inches square, Ans.
5. $18 \div .225079 = 79.97$ inches, Ans.
6. 20 rods $= 330$ feet; $330 \div .225079 = 1466.15$ feet, Ans.

(ART. **645**, p. **432.**)

1. $86.2 \div 2.155 = 40$ inches, Ans.

(ART. **647**, p. **433.**)

1. $24 \times 18 \times .785398 = 339.2919$, Ans.

2. $33\frac{5}{12} \times 20\frac{1}{4} \times .785398 = 531.469$sq. feet $= 59$sq. yd. 67sq. inches, Ans.

SOLIDS.

(ART. **650**, p. **433.**)

1. $23 + 34 + 19 = 76$ inches $= 6\frac{1}{3}$ feet, the perimeter; $6\frac{1}{3}$ $\times 13 = 82.33$ feet, the area of the sides; by Art. **613**, $38 \times 15 \times 4 \times 19 = 43320$; $\sqrt{43320} = 208.1$ inches, the area of one of the ends; $208.1 \times 2 = 416.2$; 416.2 $\div 144 = 2.89$, area of both ends in feet;

$$
\begin{array}{l}
82.33, \text{ area of sides;} \\
\underline{2.89, \text{ area of the ends;}} \\
85.22 \text{ feet, surface of the prism, Ans.}
\end{array}
$$

2. $33 \times 5 = 165$ inches $= 13\frac{3}{4}$ feet, the perimeter; $13\frac{3}{4} \times 14$ $= 192.5$ feet, area of the sides; and, $33^2 \times 1.720477 \times$ $2 = 3747.19$sq. inches $= 26.02$sq. feet, the area of the two ends; then, $192.5 + 26.02 = 218.52$sq. feet, surface of the prism, Ans.

3. 57in. $= 4\frac{3}{4}$ feet; $4\frac{3}{4} \times 13 = 61.75$sq. feet, the convex surface of cylinder; $(4\frac{3}{4})^2 \times .079577 \times 2 = 3.59$sq. ft., area of the two ends; then, $61.75 + 3.59 = 65.34$sq. ft., surface of cylinder, Ans.

4. 1 acre $= 43560$sq. feet; $1\frac{3}{4} \times 3.14159 = 5.49778$ feet, the circumference of the cylinder; $5.49778 \times 5\frac{1}{4} = 28.$ $.86334$, convex surface of the cylinder; $43560 \div 28.86$- $334 = 1509.18$ times, Ans.

5. $16 \times 10 \times 4 = 640$sq. feet; $640 \div 9 = 71\frac{1}{9}$sq. yards, Ans.

(ART. **651**, p. **434.**)

1. $2.5^2 - 1.25^2 = 4.6875$; $\sqrt{4.6875} = 2.16506 =$ the per-
   pendicular of the triangle marked by the end of the
   prism; and 2.16506×1.25 (half the base) $= 2.706 =$
   area of the end of the prism; $2.706 \times 12 = 32.47$ cubic
   feet, Ans.

2. (Art. **613**.) $6 \times 1 \times 2 \times 3 = 36$; $\sqrt{36} = 6 =$ area of
   the end; then, $6 \times 10 = 60$cu. feet, Ans.

3. $3\frac{1}{6} \times 2\frac{2}{3} \times 2\frac{1}{2} = 21\frac{1}{3}$cu. feet, Ans.

4. $6^2 \times .079577 \times 9 = 25.78$cu. feet, Ans.

(ART. **656**, p. **435.**)

1. $2\frac{3}{4} \times 2\frac{2}{3} = 7\frac{1}{3}$ feet, area of the base; $2\frac{2}{3} \times 4 = 10\frac{2}{3}$ feet,
   the perimeter of the base; $7\frac{1}{3} + 10\frac{2}{3} \times \dfrac{3\frac{3}{4}}{2} = 27\frac{1}{8}$sq.
   feet area, Ans.

2. $9 \times \frac{20}{2} = 90$ feet, Ans.

(ART. **657**, p. **435.**)

1. $(2\frac{1}{2})^2 \times .785398 \times 12_{\frac{5}{3}\cdot} = 20.45$, Ans.

2. By Art. **613**, $9 \times 4 \times 3 \times 2 = 216$; $\sqrt{216} = 14.69693$,
   the area of the base; $14.69693 \times 14_{\frac{5}{3}\cdot} = 71.035$cu.
   feet, Ans.

(ART. **658**, p. **435.**)

(1.)

$3 \times 5 = 15$, perimeter of smaller end;

$5 \times 5 = 25$, perimeter of larger end;

$\overline{40} \times 5 = 200 \qquad =$ surface of the sides;

$3^2 \times 1.720477 = 15.4842 =$ surface of smaller end;

$5^2 \times 1.720477 = \underline{43.0119} =$ surface of larger end;

258.4961 inches, surface of the frus-
[tum, Ans.

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(2.)

$3\frac{7}{12} \times 3.14159 = 11.2573$, circumference of larger end;

$1\frac{11}{12} \times 3.14159 = 6.02138$, circumference of smaller end;

$\overline{17.27868} \times 4\frac{1}{2} = 77.75406 = $ convex

[surface of the frustum;

77.75406, convex surface;

$(4\frac{3}{12})^2 \times .785398 = 10.08472$, surface of larger end;

$(2\frac{7}{12})^2 \times .785398 = 2.885246$, surface of smaller end;

$\overline{90.724026}$ sq. feet, Ans.

(Art. **659.** p. **435.**)

1. $27 \times 27 = 729$, area of larger end; $16 \times 16 = 256$, area of smaller end;

   $729 \times 256 = 186624$; $\sqrt{186624} = 432$; $432 + 256 +$

   $729 = 1417$; $1417 \times \frac{18\frac{2}{3}}{3} = 8816.888$; $8816.888 \div$

   $144 = 61.228$ cubic feet, Ans.

2. $2^2 \times .7854 = 3.1416$, area of larger end;

   $1^2 \times .7854 = .7854$, area of smaller end;

   $3.1416 \times .7854 = 2.46741264$; $\sqrt{2.46741264} = 1.5708$;

   1.5708,
   3.1416, area of larger end;
   .7854, area of smaller end;

   $\overline{5.4978} \times \frac{4.0}{3} = 73.304$ cubic feet, Ans.

(Art. **666,** p. **436.**)

1. $24 \times (24 \times 3.14159) = 1809.55$ sq. in., Ans.

2. $7957\frac{3}{4} \times 25000 = 198943750$, Ans.

(Art. **667,** p. **436.**)

1. $12^3 \times .523598 = 904.78$, Ans.

2. $25000 \times .31831 = 7957.75$, diameter; $7957.75^3 \times .5236$
   $= 263858149120.06886875$, Ans.

(ART. **668**, p. **437**.)

1. $12\frac{1}{2} \times 3.14159 \times 2 = 78.54$sq. feet, Ans.
2. $7970 \times 3.14159 \times 2143.623553 = 53673229.81$sq. miles, Ans.

(ART. **669**, p. **437**.)

1. $(3^2 + (4\frac{1}{2})^2 \times 3) \times 3 \times .5236 = 109.56$cu. feet, Ans.
2. $(9^2 + 10^2 \times 3) \times 9 \times .5236 = 1795.42$cu. feet. Ans.

(ART. **670**, p. **437**.)

1. $(21 + 4) \times 4 \times 9.8696 = 986.96$sq. in., Ans.

(ART. **671**, p. **437**.)

1. $(25 + 5) \times 5^2 \times 2.4674 = 1850.55$cu. in., Ans.

(ART. **672**, p. **437**.)

1. $20^2 \times 32 \times .523598 = 6702.05$cu. in., Ans.
2. $38^2 \times 48 \times .523598 = 36291.62$cu. feet, Ans.

(ART. **674**, p. **438**.)

1. $24 \times \frac{2}{3} = 16$ feet, Ans.
2. $30 \times 1\frac{1}{3} = 40$ feet, Ans.
3. $(26 + 14) \div 2 = 20$; $30 \times 20 \div 12 = 50$ feet, Ans.

(ART. **675**, p. **438**.)

1. $3 \times 5 \times 15 \times 3 = 675$; $675 \div 12 = 56\frac{1}{4}$ feet, Ans.
2. $2 \times 6 \times 10 \times 20 \div 12 = 200$ feet, Ans.
3. $10 \times 17 \times 20 \div 12 = 283\frac{1}{3}$ feet, Ans.

(ART. **676**, p. **438**.)

1. $30 \times 10^2 \div 144 = 20\frac{5}{6}$ feet, Ans.
2. $50 \times 14^2 \div 144 = 68\frac{1}{18}$ feet, Ans.
3. $90 \times 30^2 \div 144 = 562\frac{1}{2}$ feet, Ans.

(Art. **679**, p. **439**.)

1. $27 + (8 \times .70) = 32.6$; $32.6^2 \times 45 \times .0034 = 162.602$ wine gallons, Ans.

2. $30 + (8 \times .65) = 35.2$; $35.2^2 \times 42 \times .0034 = 176.9349$ wine gallons, Ans.

3. Consider the tub as the frustum of a cone, and apply the rule in Art. **659**; by this rule get the contents of the tub in inches, and change the inches to their equivalent in gallons;

$30^2 \times .7854 = 706.86$, area of smaller end;

$40^2 \times .7854 = 1256.64$, area of larger end;

$1256.64 \times 706.86 = 888268.5504$; $\sqrt{888268.5504} = 942.48$; $942.48 + 706.86 + 1256.64 = 2905.98$; $2905.98 \times \frac{5.0}{3} = 48433$ cubic inches; $48433 \div 231 = 209.66\frac{2}{3}$ liquid gallons, Ans.

4. $\dfrac{10 \times 5 \times 4 \times 1728}{231} = 1496\frac{8}{77}$ wine gallons, Ans.

5. $\dfrac{12 \times 6 \times 2 \times 1728}{282} = 882\frac{18}{47}$ beer gallons, Ans.

6. $\dfrac{15 \times 5 \times 7 \times 1728}{2150.42} = 421.8$ bushels, Ans.

TONNAGE.

(Art. **681**, p. **440**.)

1. $191\frac{6}{12} = \frac{2298}{12}$; $36\frac{5}{12} = \frac{437}{12}$; $\frac{2298}{12} - \frac{2}{5}$ of $\frac{437}{12} = \frac{10179}{60}$; $\frac{10179}{60} \times \frac{437}{12} \times \frac{437}{24} \times \frac{1}{95} = 1184\frac{427}{32000}$ tons, Ans.

2. $184\frac{6}{12} - \frac{2}{5}$ of $38\frac{11}{12} = \frac{2662}{60}$; $\frac{2662}{60} \times 38\frac{11}{12} \times 19\frac{11}{24} \times \frac{1}{95} = 1284\frac{22092947}{52472000}$ tons, Ans.

3. $195\frac{2}{12} - \frac{2}{5}$ of $39\frac{4}{12} = 171\frac{17}{30} \times 39\frac{4}{12} \times 19\frac{8}{12} \times \frac{1}{95} = 1397\frac{183}{12825}$ tons, Ans.

4. $78 - \frac{2}{5}$ of $21 = 65.4$; $65.4 \times 21 \times 9 \times \frac{1}{95} = 130\frac{53}{475}$ tons, Ans.

5. $141 \times 30 \times 15 \times \frac{1}{95} = 667\frac{17}{19}$ tons, Ans.

6. $479 - \frac{2}{5}$ of $80 = 431$; $431 \times 80 \times 40 \times \frac{1}{95} = 14517\frac{17}{19}$ tons, Ans.

MISCELLANEOUS QUESTIONS.

(Page 441.)

1. $\frac{2}{7} \times \frac{5}{9} = \frac{10}{63}$; $1 - \frac{10}{63} = \frac{53}{63}$, Ans.

2. As there is a son and a daughter, the son will have $\frac{4}{7}$ of the estate, the wife $\frac{2}{7}$, and the daughter $\frac{1}{7}$. If there had been only a daughter, her share would have been $\frac{2}{3}$; consequently she loses $\frac{2}{3} - \frac{4}{7} = \frac{8}{21}$. Hence

$$\frac{8}{21} : \frac{7}{21} : : \$2400 : \$2100, \text{ Ans.}$$

3. From the conditions of the question, it will readily be seen that it was a little more than half-past 5. At $5\frac{1}{2}$ o'clock the minute-hand was at 6, and the hour-hand half-way between 5 and 6, or $2\frac{1}{2}$ spaces from 6. The minute-hand moves twelve times as fast as the hour-hand; hence, while the minute-hand was moving from 6 to the required position, the hour-hand moved $\frac{1}{12}$ as far, and was then as far from the 6 point as the minute-hand was beyond. Therefore, the sum of the spaces passed over by the hour and minute hands $= 2\frac{1}{2}$ minute spaces. Hence $2\frac{1}{2}$ m. $= \frac{13}{12}$ of the required time beyond half-past 5, and $\frac{13}{12} = 2$ m. $18\frac{6}{13}$ s. 5h. 30m.$+$2m. $18\frac{6}{13}$ sec. $=$ 5h. 32m. $18\frac{6}{13}$ sec., Ans.

(4.)

```
    6)97deg. 55m. 7fur. 35rd. 4ft. 6in.(16deg.
      96
      ──
       1
      69⅙
    6)124⅙(20m.           6)208⅓(34rd.
      120                   204
      ────                  ────
        4⅙                    4⅓
        8                    16½
    6)40⅓(6fur.           6)75½(12ft.
      36                    72
      ───                   ───
        4⅓                    3½
       40                    12
    ──────                6)48(8in.
      208⅓                  48
```

Ans. 16deg. 20m. 6fur. 34rd. 12ft. 8in.

5. $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$; $\frac{1}{3} = \frac{4}{12}$; $\frac{1}{4} = \frac{3}{12}$;

$\quad \frac{7}{12} : \frac{4}{12} :: \$100,000 : \$57,142\frac{6}{7}$, A's part, $\left.\vphantom{\begin{matrix}a\\b\end{matrix}}\right\}$ Ans.

$\quad \frac{7}{12} : \frac{3}{12} :: \$100,000 : \$42,857\frac{1}{7}$, B's part,

6. If the first man's share be subtracted from the whole, there will remain $\frac{15}{8} - \frac{7}{8} = \frac{11}{8}$; and $\frac{7}{8}$ of $\frac{11}{8} = \frac{77}{324}$ = the second son's share. And $\frac{7}{18} - \frac{77}{324} = \frac{49}{324}$ = difference of their legacies. $\frac{7}{18} = \frac{126}{324}$; $\frac{126}{324} + \frac{75}{324} = \frac{201}{324}$ = legacy of both sons. Hence $\frac{324}{324} - \frac{201}{324} = \frac{123}{324}$ = wife's legacy. Therefore

$\qquad \frac{49}{324} : \frac{123}{324} :: 257£.\ 3s.\ 4d. : 635£.\ 0s.\ 10\frac{3}{4}d.$, Ans.

7. $63 \times 12 \times 12 \times 1000 \times 3 = 27216000$; $27216000 \div 16 = 1701000$lb.; $1701000 \div 2240 = 759\frac{3}{8}$ tons, Ans.

8. 4ft. = 48in.; 6in. $\times$ 2 = 12in.; 48 — 12 = 36in.; 36 $\div$ 2 = 18in.; 18 + 12 = 30in.; 48in. : 30in. :: 200lb. : 125lb., Ans.

$\quad$ 48 — 30 = 18in.; 48in. : 18in. :: 200lb. : 75lb., Ans.

9. 25ft. 4in. = 304in.; 4ft. 5in. = 53in.; 3ft. 5in. = 41in.; $53 \times 41 = 2173$; $53 - 41 = 12$in.; $12 \times 12 = 144$; $144 \div 3 = 48$; $2173 + 48 = 2221$; $2221 \times .785398 = 1744.368958$; $1744.368958 \times 304 \times 8 = 4242305.305856$; $4242305.305856 \div 1728 = 2455.037792 =$ cubic feet in the pillars. $2455.037792 \times 3000 = 7365113.376 =$ weight in ounces; $7365113.376 \div 16 = 460319.586$lbs.; $460319.586 \div 2240 = 205.49981+$ tons, Ans.

10. If $\frac{3}{7}$ of a certain sum be taken, and \$410 be left, it is evident that \$410 is $\frac{4}{7}$ of that sum, which is $\overline{410 \times 7} \div 4 = \$717\frac{1}{2}$. Now, if \$717$\frac{1}{2}$ remain of a certain quantity after $\frac{1}{4}$ be subtracted, it is certain that the number from which it is taken is $\frac{4}{3}$ of \$717$\frac{1}{2}$ = \$956.66$\frac{2}{3}$, Ans.

11. $\frac{1}{4} : \$15.60 :: \$100 : \$6240 =$ sum remitted; $\$96 : \$100 :: \$6240 : \$6500 =$ value of goods sold; $\$6500 - \$6240 = \$260 =$ commission, Ans.

12. $\$107.50 : \$100 :: \$9675 : \$9000$; $\frac{9}{40} \times \$9000 = 2025£$. sterling = the bill; $\$100 - \$0.25 = \$99.75$;

$100 : $99.75 : : $9675 : $9650.81¼; $102 : $100 : : $9650.81¼ : $9461.58₃²₄ for investment, Ans.

13. $15 \times 30 = 450$; $15 \times 15 = 225$; $225 \div 3 = 75$; $450 + 75 = 525$; $525 \times 220 = 115500 =$ contents of the whole monument, and from this we deduct the contents of the cylinder. $15 \times 11 = 165$; $4 \times 4 = 16$; $16 \div 3 = 5⅓$; $165 + 5⅓ = 170⅓$; $170⅓ \times .785398 = 133.779459⅓$; $133.779459⅓ \times 220 = 29431.481053⅓ =$ contents of the cylinder; $115500 - 29431.481953⅓ = 86068.51894+$ cubic feet of the monument, Ans.

(14.)

$\frac{4}{12} \times 1 + \frac{4}{12} \times \frac{1}{2} + \frac{4}{12} \times \frac{1}{3} = \frac{22}{36}$, A's product;
$\frac{4}{12} \times \frac{1}{2} + \frac{4}{12} \times \frac{1}{3} = \frac{10}{36}$, M's product;
$\frac{4}{12} \times \frac{1}{3} = \frac{4}{36}$, P's product;

$\overline{\frac{36}{36}}$, sum of the products.

$\frac{36}{36} : \$300 : : \frac{22}{36} : \$183.33⅓ =$ A pays, $\Big\}$
$\frac{36}{36} : \$300 : : \frac{10}{36} : \$ 83.33⅓ =$ M pays, $\Big\}$ Ans.
$\frac{36}{36} : \$300 : : \frac{4}{36} : \$ 33.33⅓ =$ P pays, $\Big\}$

(15.)

A $20 \times \$132 = \$2640$
B $25 \times \$120 = \$3000$ $85 \times \$6 = \$510.$
C $40 \times \$100 = \$4000$
$\overline{\quad 85 \qquad\qquad \$9640}$

$\$9640 : \$510 : : \$2640 : \$139\frac{16}{24}¼$, A receives, $\Big\}$
$\$9640 : \$510 : : \$3000 : \$158\frac{12}{24}¾$, B receives, $\Big\}$ Ans.
$\$9640 : \$510 : : \$4000 : \$211\frac{14}{24}⅔$, C receives, $\Big\}$

(16.)

$\$100 \times \$5.00 = \$500$ given for the flour;
$\$500 \times \$0.20 = \$100$ gained on the flour;
$\overline{\qquad\qquad \$600}$

$\$100 \times \$0.03,0½ = \$3.05$ bank interest of $100 for 6 months; $\$100 - \$3.05 = \$96.95 : \$100 : : \$600 : \$618\frac{180}{1939}$; $\$618\frac{180}{1939} \div 100 = \$6.18\frac{180}{1939}$, Ans.

17. We first find the number of square feet in an acre. $160 \times 272\frac{1}{4} = 43560$ feet. If we extract the square root of this number, we obtain the side of a square field that will contain an acre; thus, $\sqrt{43560} = 208.712+$ feet. We now divide this number by 3.5 feet, and obtain $59.632+$, the number of divisions in the first row. We perceive, therefore, that there will be 60 hills, there being one more hill than divisions. Now, if we divide $208.712+$ by 59, the quotient will be 3.537 feet; that is, the hills in the first row may be 3.537 feet apart, instead of 3.5 feet. Thus, our first row will contain 60 hills, which will be 3.537 feet apart. Our next row will contain but 59 hills, the hills being planted in the quincunx order, thus:

To find the distance between the rows, we square $3.5 = 12.25$; we then take half of $3.537 = 1.768$, which we square $= 3.125824$; we subtract this last number from 12.25, and obtain 9.124176. The square root of this number is 3.0206 feet, equal the distance between the rows. Now, if we divide $208.712+$ by 3.0206, we obtain $69+$; therefore, the number of rows will be 70. To obtain the number of hills in the field, we multiply 70 by $60 = 4200$. But, as there are 70 rows, and as half of the rows contain only 59 hills, we subtract 35 from 4200. Thus, $4200 - 35 = 4165$ hills, Ans.

18. $\$300 : \$700 : : 20$ months $: 46\frac{2}{3}$ months, Ans.

19. $\$1500 \div 150 = 10$; $10 + 2 = 12$ children. $\$1500 \times 2 = \$3000$; $\$3000 \div 3 = \$1000$; $\$3000 + \$1000 = \$4000$; $\$4000 \div 4 = \$1000$; $\$4000 + \$1000 = \$5000$; $\$5000 \times 2 = \$10,000$, Ans.

20. $7 - 5 = 2$ miles which B gains each day, and he will have to gain 80 miles before he overtakes A. 2 miles : 80 miles : : 1 day : 40 days, the time which it takes B to

overtake A. And as B travels 7 miles each day, he will have to go, before he overtakes A, $40 \times 7 = 280$ miles, Ans.

21. 16lb. : : 80lb. : : 24.4in. : 122 cubic inches of lead. $\frac{1}{4} \times 2 = \frac{1}{2}$in.; 1in. $+ \frac{1}{2}$in. $= 1.5$ inches; $1.5 \times 1.5 = 2.25$; $2.25 \times .785398 = 1.7671455 =$ area of a section of the pipe. From this we subtract the area of a section of the calibre of the pipe. $1 \times .785398 = .785398$; $1.7671455 — .785398 = .9817475$; $122 \div .9817475 = 124.26+$ inches, $= 10.35+$ feet, Ans.

22. $.785398 \times 2 \times 2 = 3.141592$; $\frac{3}{4} \times \frac{3}{4} \times .785398 = .441786$; $3.141592 — .441786 = 2.699806$; $2.699806 \times 8 = 21.598448$; $\frac{3}{4} \times 2 = \frac{6}{8} = \frac{3}{4}$; $\frac{3}{4} + \frac{3}{4} = 1.5$; $1.5 \times 1.5 = 2.25$; $.785398 \times 2.25 = 1.7671455$; $1.7671455 — .441786 = 1.3253595$; $21.598448 \div 1.3253595 = 16.29+$in., Ans.

23. Let $100 represent the *relative* value of what D pays; then $100 \times 1.08 = 108 will represent what C pays; $108 \times 1.10 = 118.80$, what B pays; and $118 \times 1.08 = 133.056, what A pays. $100 + $108 + $118.80 + $133.056 = 459.856.

$459.856 : $100 : : $100 \qquad : $21.74\frac{1796}{2871}$, D pays.

$459.856 : $100 : : $108 \qquad : $23.48\frac{1812}{2871}$, C pays.

$459.856 : $100 : : $118.80 : $25.83\frac{1927}{2871}$, B pays.

$459.856 : $100 : : $133.056 : $28.93\frac{2281}{2871}$, A pays.

$20 \times 20 \times 20 = 8000$; $459.856 : $133.056 : : 8000 : 2314.742006$; $\sqrt[3]{2314.742006} = 13.22+$ft., which A takes; $459.856 : $251.856 : : 8000ft. : 4381.47594ft.; $\sqrt[3]{4381.47594} = 16.36+$ft.; $16.36 — 13.22 = 3.14+$ ft. which B takes. $459.856 : $359.856 : : 8000ft. : 6260.324971ft. $\sqrt[3]{6260.324971} = 18.42$ft.; $18.42 — 16.36 = 2.06+$ft. which C takes. $20 — 18.42 = 1.58+$ft. which remains for D.

24. As B paid 20 per cent. more than A, and 10 per cent. less than C, we obtain their shares paid as follows:

21

For each dollar A pays, B pays $1.20, and C pays $1.33⅓;
  $1 + $1.20 + $1.33⅓ = $3.53⅓. Therefore,

   $3.53⅓ : $10.60 : : $1.00   : $3.00, A paid.
   $3.53⅓ : $10.60 : : $1.20   : $3.60, B paid.
   $3.53⅓ : $10.60 : : $1.33⅓ : $4.00, C paid.

As A paid $3.00, his share of the stone will be

$$\frac{300}{1060} = \frac{15}{53}; \quad B's, \frac{360}{1060} = \frac{18}{53}; \quad C's, \frac{400}{1060} = \frac{20}{53}.$$

$65^2 = 4225$, square of the diameter of the stone.

$3^2 = 9$ in., square of the place for the axle.

$4225 - 9 = 4216$, to be divided among A, B, and C.

$4216 \times \frac{15}{53} = 1193.20$, A's part.

$4216 \times \frac{18}{53} = 1431.84$, B's part.

$4216 \times \frac{20}{53} = 1591$, C's part.

$4225 - 1193.20 = 3031.80$; $\sqrt{3031.80} = 55$ inches.

$65 - 55 = 10$ in.; $10 \div 2 = 5$ inches, A grinds off.

$3031.80 - 1431.8 = 1600$; $\sqrt{1600} = 40$ inches.

$55 - 40 = 15$; $15 \div 2 = 7\frac{1}{2}$ inches, B grinds off.

$1600 - 1591 = 9$; $\sqrt{9} = 3$ inches.

$40 - 3 = 37$; $37 \div 2 = 18\frac{1}{2}$ inches, C grinds off.

A grinds off 5 inches, B 7½ in., and C 18½ in., Ans.

NOTE. — In the solution of this problem we have omitted small fractions.

25. It is evident that in every case the drawing off of one gallon
     from the cask full leaves in it $\frac{9}{10}$ of its previous contents.
     Hence the quantity of wine left the first day is $\frac{9}{10}$ of 10
     gallons; the second day, $\frac{9}{10}$ of that; and so on, till at the
     20th day it is only 10 gallons multiplied by the twen-
     tieth power of $\frac{9}{10}$; and if this quantity be taken from 10
     gallons, the remainder will be the quantity of water. By
     similar reasoning it would be shown that the quantity of
     water contained in the cask, at the end of the second
     period of twenty days, would be equal to the quantity
     last mentioned, multiplied also by the twentieth power of
     $\frac{9}{10}$. Now, the twentieth power of $\frac{9}{10}$, or .9, is .121576-
     65459. ($.9^4 = .6561$; $.9^8 = .43046721$; $.9^8 \times .9^8 \times$

$.9^4 = .9^{20} = .12157665459$.)   The product of this by 10 taken from 10, the remainder is 8.7842334541, the quantity of water in the cask at the end of 20 days; and the product of this by $.9^{20}$ is 1.0679577+ gallons, or more than a gallon and half-pint, Ans.

26. $18.5 \times 18.5 \times 18.5 \times 8 = 50653$; $\sqrt[3]{50653} = 37$in. wide; $8 \times 8 \times 8 \times 8 = 4096$; $\sqrt[3]{4096} = 16$in. deep, Ans.

27. As the metal is 1in. thick, the diameter of the inner sphere is 3in.; $5 \times 5 \times 5 \times .5236 \times \frac{128}{528} = 16.8861$lb., weight of the shell, if it were solid iron; $3 \times 3 \times 3 \times .5236 \times \frac{128}{528} = 3.6473976$lb., weight of the inner sphere, if it were iron; $16.8861 - 3.6473976 = 13.2387+$lb., Ans.

28. As the two hands had precisely changed positions, they together had passed round through all the spaces of the dial-face; but, as the minute-hand always goes through 60 spaces while the hour-hand goes through 5, both going through 65, therefore $65 : 5 : : 60$ (the distance passed over by both hands) : $4\frac{8}{13}$min. or spaces, passed over by the hour-hand, and which is also the distance the minute-hand was in advance of the hour-hand.   But at 2 o'clock the hour-hand was 10 minutes in advance of the minute-hand; consequently the minute-hand had gained 10min. $+ 4\frac{8}{13} = 14\frac{8}{13}$min. on the hour-hand; then, since the minute-hand always gains 55min. in 60min., how long was it in gaining $14\frac{8}{13}$min.?

$55 : 60 : : 14\frac{8}{13} : 15\frac{143}{143}$min. $= 15$ min. $56\frac{92}{143}$sec. after 2, Ans.

29. $20 \times 20 = 400$; $400 \div 3 = 133.\dot{3}$; $\sqrt{133.\dot{3}} = 11.5469$; $\overline{11.5469}^3 = 1539.58+$cu. in., Ans.

30. $90 \times 40 = 3600$; $\sqrt{3600} = 60$lb., true weight, Ans. $90 - 60 = 30$lb.; $60 - 40 = 20$lb.; 30lb. : 20lb. : : 3ft. : 2ft.; that is, the arms of the scales are to each other as 2ft. to 3ft., Ans.

31. Both wheels being of the same height, and the outer

wheel making two turns while the inner one makes only one turn, it will follow that the outer ring will be twice the circumference of the inner ring. The distance between the rings being 5 feet, and the circumferences of circles being as their diameters, it will also follow that the diameter of the inner ring will be 10 feet, and the diameter of the outer ring 20 feet. And if the diameter be 20 feet, the circumference will be 62.83+ feet, Ans.

32. $72 \times 72 \times 3.141598 = 57001$d. $= 237£.$ 10s. 1d., Ans.

33. The annexed diagram may represent the conical glass, A B C being the cone, and F D G H a globe or sphere immersed in it. If A B be 5 inches, A D will be 2.5 inches, because A D is half of A B. A D C is a right-angled triangle; therefore the side A C may be found; thus, $\sqrt{A\,D^2 + D\,C^2} =$ A C; $\sqrt{2.5 \times 2.5 + 6 \times 6} = 6.5 =$ A C. Because A D E F is a regular figure, and the angles A D E and A F E being equal, each being a right angle, and the sides D E and F E being also equal, because they are radii of the circle D F H G, the sides A D and A F are also equal. A D is 2.5 inches; A F is also 2.5 inches. If A C be 6.5 inches, and A F 2.5 inches, F C will be 4 inches; $6.5 - 2.5 = 4$ inches. Then, by similarity of triangles (see page 389), C D : D A :: C F : F E; 6in. : 2.5in. :: 4in. : 1⅔in. = F E. If F E be 1⅔in., F G will be 3⅓in. = ⅒in, because F G is the diameter of the sphere, and F E the radius, or semi-diameter. By mensuration of solids (see pages 435 and 436), we find the contents of the cone and sphere in the following manner:
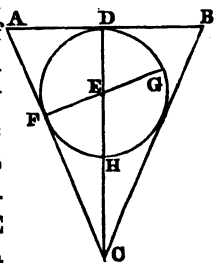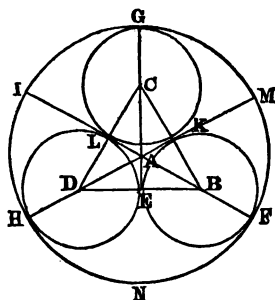
$5 \times 5 \times .785398 \times 2 = 39.2699$in. = contents of the cone;

$\frac{10}{3} \times \frac{10}{3} \times \frac{10}{3} \times .5236 = 19.3925$in. = contents of the sphere;

19.8774in. = the cubic inches of water

that will remain in the cone after the sphere is immersed. Having taken it for "granted" that cones, spheres, and all similar bodies, are to each other as the cubes of their homologous sides, we say, As the quantity of water it requires to immerse the sphere in the *given* cone is to the cube of the diameter of the sphere, so is any other quantity of water in the conical glass to the cube of the diameter of a sphere that may be immersed in it. Now, the quantity of water given to immerse the *required* sphere is $\frac{1}{5}$ of the contents of the conical glass $= \frac{39.2699}{5} = 7.85398$ cubic inches. The cube of the diameter of the *given* sphere is $\frac{10}{3} \times \frac{10}{3} \times \frac{10}{3} = \frac{1000}{27}$in. Therefore

$$19.8774\text{in.} : \frac{1000}{27}\text{in.} : : 7.85398\text{in.} : 14.634114529+\text{in.};$$
$$\sqrt[3]{14.634114529} = 2.445+\text{in., Ans.}$$

34. Let the larger circle of the annexed figure represent the farm of the lady, and the three smaller circles the farms of her daughters. To construct this figure, make the equilateral triangle C B D, each of whose sides is 10 rods, or inches. Bisect each of the sides D C, C B, B D, in the points L, E, K; and draw the lines I F, E G, H M, at pleasure. Upon the points C, D, B, as centres, and with the distance C L as a radius, describe the circles G L K, L H E, K E F. Upon the point A, as a centre, where the lines I F and H M intersect each other, and with the radius A G, describe the circle G I H N F M, and it will touch the peripheries of the smaller circles without cutting them. The sides of the triangle being 10, the diameter of each of the smaller circles will be 10. Because C D E is a right-angled triangle, C E $=$
$$\sqrt{\text{C D}^2 - \text{D E}^2}; \quad \sqrt{10 \times 10 - 5 \times 5} = 8.660254+;$$

21*

and as C D E and A D E are similar triangles, C E :
C D : : D E : D A; that is, 8.660254 : 10 : : 5 : 5.77-
35027 = A D. If we add H D = 5 to D A, we have
the semi-diameter of the larger circle, 5.7735027 + 5 =
10.7735027. By multiplying this last number by 2, we
have the diameter of the larger circle, 10.7735027 × 2
= 21.5470054. As the area of a circle may be found
by multiplying the square of the diameter by .785398,
therefore, by dividing the area by .785398, the quotient
will be the square of the diameter. The area of the lady's
field is 500 acres = 80000 square rods; 80000 ÷ .785398
= 101859.18, square of the diameter; √101859.18 =
319.154006+ rods = diameter of the lady's farm. To
find the diameter of each of the daughters' farms, we say,
As the diameter of the larger circle in the diagram is to
the diameter of one of the smaller circles in the diagram,
so is the diameter of the lady's farm to the diameter of
either of her daughters' farms. 21.5470054rd. : 10rd.
: : 319.154006rd. : 148.119889+rd. = diameter of the
daughters' farms; and the distance of their houses from
each other, 148.119889 × 148.119889 × .785398 =
17231.2406+ square rods in each of the daughters'
farms; 17231.24+rd. ÷ 160 = 107A. 2R. 31.24+
rods = acres, &c., in each of their farms; 107A. 2R.
31.24rd. × 3 = 323A. 0R. 13.72rd., amount of the
3 daughters' farms; 500A. — 323A. 0R. 13.72rd. =
176A. 3R. 26.28rd. the lady retained. To find the dis-
tance of the lady's dwelling-house from those of her
daughters, we subtract the semi-diameter of either of the
daughters' farms from that of the lady's; thus, 319.154+
rd. ÷ 2 = 159.577+rd.; 148.119+rd. ÷ 2 = 74.059+
rd.; 159.577+rd. — 74.059+rd. = 85.518+rd.

We therefore find that each daughter's farm contained
107A. 2R. 31.22p. The mother retained 176A. 3R.
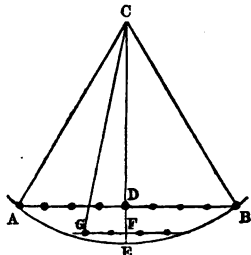26.34p. The distance from one daughter's house to the

other was 148.119817+ rods. The mother's dwelling-house was distant from her daughters, 85.51+ rods, Ans.

This question can also be solved by using the principle laid down in Art. **645.**

35. The pupil, to understand this problem, will first obtain the number of feet in the diameter of the garden; $10 \times 16.5 = 165$ feet; $165 - 5 = 160$. The trees are, therefore, to be set on a piece of ground 160 feet in diameter. Let the pupil place 1 tree in the centre of the garden; around this let him place 6 other trees, at the distance of 10 feet from each other; he will then perceive that they stand in a hexagonal form. Let him enlarge this hexagon by placing another row of trees around it at the distance of 10 feet each; and this will require 12 additional trees. If we examine this hexagon, we shall find that each side of it contains 3 trees. Let us enlarge this hexagon, by placing another row of trees around it, and we shall find it will require 18 trees, and that each side of the hexagon contains 4 trees. We continue thus to enlarge the hexagon, until we have set 8 rows round the centre tree. Each side of the hexagon will then contain 9 trees. To compute the number of trees in the hexagon, we find the number of trees that compose the periphery of the first hexagon to be 6 trees, and the number that compose the periphery of the larger hexagon to be 48. We therefore add 6 to 48, and multiply the sum by the half of 8 = 4; thus, $6 + 48 = 54$; $54 \times 4 = 216$. To this we add the tree in the centre, $216 + 1 = 217$. If we now examine our figure, we find we can set 4 more trees at the base of each side of the hexagon, within the limits of the prescribed field. Therefore, 4 times $6 = 24$, to be added to 217; thus $217 + 24 = 241$ trees, Ans.

NOTE. — As the radius of the prescribed limits is 80 ft., $80 \div 10 = 8 =$ number of hexagons.

To prove that there can be 4 trees, and *only* 4, placed beyond each side, let A B represent one side of the outer hexagon, C being the centre of the garden, and the arc A E B the prescribed limit, 2½ feet from the outside of the garden. Draw C E perpendicular to A B, and it will also bisect it. $\overline{C\ D}^2 = \overline{C\ A}^2 - \overline{A\ D}^2 = 6400 - 1600 = 4800$; $\sqrt{4800} = 69.28 + = C\ D.$ C E $- C\ D = 80 - 69.28 = 10.72 = D\ E$; hence it is evident that there can be another row placed below A B. To find the distance of this row from A B, we have to find the altitude of an equilateral triangle, each side of which is 10 ft. $10^2 - 5^2 = 75$; $\sqrt{75} = 8.66 + = D\ F.$ $69.28 + 8.66 = 77.94 + = C\ F.$ Having placed 4 trees on this row, we wish to ascertain whether the one at G is within the limit. $\overline{C\ G}^2 = \overline{C\ F}^2 + \overline{F\ G}^2 = 6075 + 225 = 6300$; $\sqrt{6300} = C\ G.$ As this is less than 80, G is *within* the limit. If we add one more tree on a line with F G, its distance from C $= \sqrt{6075 + 625} = \sqrt{6700}$, which is greater than 80, and consequently the tree would be *without* the limit. Hence there can be *only* 4 trees added on each side.

36. $90s. = 1080d.$; $3s.\ 6d. = 45d.$ As A would reap the field in 9 days, he would in 5 days reap $\frac{5}{9}$ of the field; therefore, $1 - \frac{5}{9} = \frac{4}{9} =$ the part of the field which B and C reap; $\frac{45}{1080} = \frac{9}{216}$; $\frac{4}{9} \div \frac{9}{216} = \frac{864}{81}$; $\frac{864}{81} - \frac{81}{81} = \frac{783}{81}$; $\sqrt{81} = 9$; $783 \div 9 = 87$; $87 \div 2 = 43.5$; $(43.5)^2 = 1892.25$; $1892.25 - 1080 = 812.25$; $\sqrt{812.25} = 28.5$; $43.5 - 28.5 = 15$ days = the time B would reap the field. We therefore perceive that A would do $\frac{5}{9}$ of the work, and B $\frac{5}{15}$ of it, in 5 days ; $\frac{5}{9} + \frac{5}{15} = \frac{8}{9}$ of the work would be performed by A and B in 5 days. Therefore, $1 - \frac{8}{9} = \frac{1}{9}$ would be performed by C in 2 days, or $\frac{1}{18}$ in 1 day. And if $\frac{1}{18}$ of it be reaped in 1 day, it is evident that it would require 18 days for C to perform the whole labor. Therefore we find that B would reap the field in 15 days, and C in 18 days, Ans.

OPERATION BY ALGEBRA.

4£. 10s. =1080d.; 3s. 9d. =45d. Let $x=$ the time in which

B can do the work. Then $9 : x : : 45 : \dfrac{45x}{9} =$ the sum

which C must receive from A in part payment for his

labor. Then $45 + \dfrac{45}{x} =$ the money received by C for

his 2 days' labor. Now, it is evident that the sum received
by C must bear the same proportion to the sum received
for the whole work, as the part of the work which he
performs bears to the whole work.

Hence $\dfrac{45 + \frac{45x}{9}}{1080} = \dfrac{x+9}{216}$ denote the part of the work per-

formed by C. Therefore $\dfrac{x+9}{216}$ work : 1 work : : 2 days :

$\dfrac{\frac{2}{x+9}}{216} = \dfrac{432}{x+9} =$ the time in which C would do the whole

work. Now, since from the question and the operation

we see that A performs $\frac{5}{9}$ of the work, B $\dfrac{x}{9}$ of it, and C

$\dfrac{x+9}{216}$ of it, it is evident that $\dfrac{5}{9} + \dfrac{x}{9} + \dfrac{x+9}{216} = 1$ work.

Or, $\dfrac{x}{9} + \dfrac{x+9}{216} = 1 - \dfrac{5}{9} = \dfrac{4}{9}$, which, being reduced and

transposed, gives $x = 15$ days $=$ the time in which B

would reap the field. $\dfrac{432}{x+9} = \dfrac{432}{24} = 18$ days $=$ the

time in which C would reap the field, Ans.

87. $4 - \frac{1}{4} = 3\frac{7}{8}$; $4 : 3\frac{7}{8} : : 40 : \frac{115}{4}$; $1\frac{1}{4}$yd. $= 24$na.; 24na.
$- 1\frac{1}{4}$na. $= 22\frac{1}{4}$na.; 24na. : $22\frac{1}{4}$na. : : $2\frac{1}{4}$yd. : $2\frac{7}{64}$yd. $=$
$\frac{135}{64}$yd.; $\frac{115}{4} \times \frac{135}{64} = \frac{20925}{256} = 81\frac{189}{256}$yd.; 5qr. $=$
$1\frac{1}{4}$yd.; $20 - 1\frac{1}{4} = 18\frac{3}{4}$yd.; $\dfrac{18\frac{3}{4}}{20} = \frac{15}{16}$; 5qr. $= 20$na.;
$20 - \frac{1}{2} = 19\frac{1}{2}$; $19\frac{1}{2} \times \frac{15}{16} = 18\frac{9}{32}$na.; $81\frac{189}{256}$yd. $=$
$1307\frac{13}{16}$na.; $1307\frac{13}{16} \div 18\frac{9}{32} = 71\frac{7}{13}$yd., Ans.

38. Let the tower at A be 30 feet high; that at B, 40; at C, 50.

First. What point F in the side A B is equally distant from the top of the tower at A and the top of that at B ?

The square of the distance from F to the top of the tower A is the square of its distance from the foot of A $+$ 900; the square of the distance from F to the top of B is the square of the distance from the foot B $+$ 1600; if the distances of F from the tops of A and B are equal, the square of its distance from the foot A is greater by 700 than the square of its distance from the foot B. A F² — B F² = 700. But A F + B F = 200; now, A F² — B F² = (A F + B F) (A F — B F), A F — B F = 3½; and therefore A F = 101¼, B F = 98¼.

Finding a similar point G on B C, we get B G = 102¼, C G = 97¾.
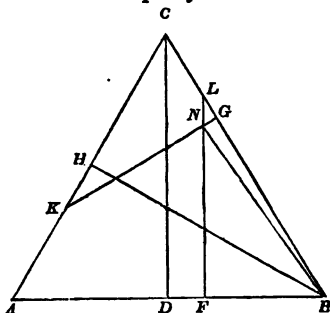
At what point L does the line F L perpendicular to A B meet B C ? Draw C D perpendicular to A B.

Then, as A and B are similarly situated with regard to C, A D = D B, and each = 100.

Then D F = 1¾; therefore C L = 3½, as D F must be ½ of C L, if D B is ½ of C B.

If C L = 3½, L G must be C G — C L = 97¾ — 3½ = 94¼. Draw G K perpendicular to B C.

Now, the foot of the ladder is on the line F L, for every point in F L is equally distant from the top of the two towers A and B. Again, it is on the line G K, for every point in G K is equally distant from the top of the towers B and C ; therefore it is on the intersection N of F L and G K.

Now, the triangle L N G is similar to C D B ; then L N is 2 times N G, and $\overline{LN}^2$ = 4 times $\overline{NG}^2$; then $\overline{LG}^2$ = 3 times $\overline{NG}^2$; $\overline{LG}^2$ = 8883.0625, then $\overline{NG}^2$ = 2961. .0208⅓.

The square of the length of the ladder = height of tower
$C^2 + C G^2 + N G^2$.

- Square of height of C, . 2500
- $C G^2$, ... 9555.0625
- $N G^2$, ... 2961.020833̇
- Square of length of laddder, 15016.083333̇
- Length of ladder, . . 122.54+

To find the distance of the foot of the ladder from the foot of each tower, we subtract the square of the height of each tower from the square of the length of the ladder. This gives

$\overline{N A^2}$ = 14116.083̇	N A = 118.811
$\overline{N B^2}$ = 13416.083̇	N B = 115.827
$\overline{N C^2}$ = 12516.083̇	N C = 111.875

SECOND SOLUTION.

A line drawn from either angle to the middle of the opposite side divides the garden into two equal right-angled triangles; and the length of this line, found in the usual way, is 172.2+ft. Draw, in the same manner, lines from the other two angles, and the three lines will intersect each other at the centre; and the garden will be divided into six equal triangles, *similar* to the first two. Then 172.2+ : 200 :: 100 : 115.6ft. = distance of the centre from the foot of either tower. A ladder placed on this centre, reaching to the top of one of the towers, will be the hypothenuse of a vertical triangle, of which the tower is the perpendicular, and 115.6ft. the base; and therefore the square of this base, + ⅓ of the sum of the squares of the three towers, will be the square of the length of a ladder, which, placed at some point, will reach to the top of each of the towers. Finally, find the distance of this point from the foot of each tower, as in the last paragraph of the first solution.

THE END.

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